A FIRST COURSE IN MATHEMATICS FOR TECHNICAL STUDENTS

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IN

MATHEMATICS

FOR TECHNICAL STUDENTS

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PREFACE.

THIS little book is intended to meet the growing demand for a simple and practical course on the rudiments of mathematics suitable for students who are preparing for a course of technical study. It is modelled on a scheme covering all the usual requirements of a First Year's Course in Preliminary Technical Classes.

The authors have endeavoured to insure that the atmosphere of the workshop should pervade the whole book. Wherever possible the problems deal with concrete quantities which refer to actual working conditions. The necessary reference which the student must make to the drawings accompanying many of the problems should form a good introduction to the art of "reading" machine drawings, plans, etc.

Graphical methods have been introduced into almost every chapter, as the necessity for the correlation of technical drawing and mathematics cannot be too strongly insisted upon.

When studying the common solids it is important that the student should *make* paper models of them. Full instructions are given in the text.

Special emphasis has been laid on the importance of approximate calculations. From the practical point of view it must be remembered that although accuracy is essential in all cases it differs in degree. The rough estimate may, in the end, be quite

as near the truth as the calculation made at leisure, for in most cases the data are only approximate to begin with.

The student should be encouraged to apply a rough check to every example he works. This rough mental estimate of a result is not only a useful check in itself, but it is invaluable as forming a habit essential to successful workshop practice.

The scheme of work will be found to cover all the requirements of the first year examination in Mathematics in the Preliminary Technical Course of the Lancashire and Cheshire Union of Institutes, the Education Committee of the County Council of the West Riding of Yorkshire, the National Union of Teachers, and other similar examining bodies.

As the authors are lecturers in the employment of the London County Council it is necessary for them in accordance with recent regulations to state explicitly that the London County Council are in no way responsible for the contents of this book and have taken no part in its publication.

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CHAPTER I.

MEASUREMENT : DECIMAL FRACTIONS.

Measurement.—If we were asked to determine the length of the line shown in Fig. 1, we should apply some kind of graduated scale to it. In this country we are accustomed to the length called an "inch," and if an inch scale were applied to this line it would reveal the fact that the length lay between two inches

Fig. 1.

and three inches (a length measured in inches, say three inches, is frequently written thus: $3''$). If the inches were subdivided into quarters we could say that the required length lay between $2\frac{1}{2}''$ and $2\frac{3}{4}''$.

Now it is readily seen that whatever be the subdivisions of the inch, it is very unlikely that they will exactly fit the line in question, and even if they did it would be unreasonable to suppose that they would meet every case which was likely to arise.

By far the most convenient way out of this difficulty is to have the inch divided into ten equal parts called "tenths." The application of such a scale to the line would show that its length was $2'' + \frac{6}{10}''$ + a small piece, less than $\frac{1}{10}''$. Now suppose we had a tenth of an inch subdivided into tenths, each of these would be equal to one hundredth of an inch. By this means the length of the small piece which was "left over" from our former

measurement could now be determined. Suppose its length were $\frac{7}{100}''$ this would bring the total length of the line to

$$2'' + \frac{6}{10}'' + \frac{7}{100}''.$$

This is about as far as we can go with the unaided eye, but if by some mechanical means (*e g.* a microscope or a micrometer) a hundredth of an inch could be subdivided into tenths (the latter being thousandths of an inch), it is more than likely that we should find that the length of the small piece just measured was not *exactly* $\frac{7}{100}''$ at all, but there was still another small piece left over (or perhaps under), this time less than $\frac{1}{100}''$. Let us suppose that there was a length of $\frac{3}{1000}''$ in excess of the last measurement. The measured length of the line now becomes

$$2'' + \frac{6}{10}'' + \frac{7}{100}'' + \frac{3}{1000}''.$$

Decimals.—Now this is a very convenient means of measurement, but we must have a simpler method of setting down a result. The length given above might be written 2·673″, which should be read "two, decimal six seven three." The dot between the two and the six simply indicates that the figure or figures on the left constitute a "whole number," while those on the right successively indicate the number of tenths, hundredths, thousandths, etc., of the unit.

Think of a number such as 444·4444. Here we have seven fours, but we know quite well that they have not all the same value, for the first has a value of 400 and the second of 40 only. Each four has only one tenth the value of its neighbour on the left hand. This rule holds good both before and after the decimal point is passed. The values might be written down thus :—

$$\begin{array}{cccccccc} 400 & 40 & 4 & & \frac{4}{10} & \frac{4}{100} & \frac{4}{1000} & \frac{4}{10000} \\ \| & \| & \| & & \| & \| & \| & \| \\ 4 & 4 & 4 & \cdot & 4 & 4 & 4 & 4 \end{array}$$

We thus see that the decimal point is no more than a dividing line between the digits whose values are not less than unity and those whose values are less than one.

The student should now make an inch scale in which at least one of the inches is divided into ten equal parts. The method of doing this is now described.

Ex. 1. *Construct a scale of inches and tenths*

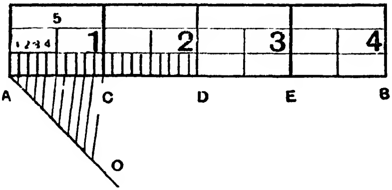

Fig. 2.

Draw a line AB 4″ long and then rule three parallel lines $\frac{1}{4}''$, $\frac{1}{2}''$ and $\frac{3}{4}''$ respectively from AB. (See Fig. 2, which is drawn to half the proper size.) Mark off with the aid of the rule the positions of the vertical lines at C, D, and E, each one inch apart. Draw through the point A a line AO of any length making an angle of about 45° with AB, take your compasses and step along this line any ten equal divisions. Join O and C and draw 9 lines parallel to OC through the divisions just made, then draw vertical lines where these sloping lines cut AC, as shown.

An inch is the unit by which small distances are measured in this country, but in many other parts of Europe the unit used is called a centimetre. By the aid of the centimetre scale on his ruler the student should now make a scale for himself one centimetre of which is divided into tenths by the same method as that just adopted in making the inch scale.

The centimetre is one unit in a system known as the **Metric System.** The standard unit of length in this system is known as a **Metre** (it is rather over a yard in length). Multiples and submultiples of this are taken and named as follows:—

kilo-metre = 1000 metres
hecto-metre = 100 metres
deca-metre = 10 metres.

METRE.

deci-metre = $\frac{1}{10}$ metre
centi-metre = $\frac{1}{100}$ metre
milli-metre = $\frac{1}{1000}$ metre.

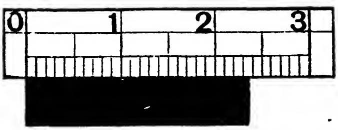

Fig. 3.

It will be noticed that the names differ only in the prefix, those printed in heavy type being most frequently employed.

Fig. 3 (which is drawn to half size) shows a scale of inches applied to a length to be measured. We see at once that

the length lies between 2·3″ and 2·4″ and this is sometimes written 2·3 +. It is very important that the student should persevere in *mentally* dividing up the last tenth into ten equal parts and so "estimating" the value of the second figure after the decimal in the above measurement. On examining the case shown in Fig. 3 it is seen that the length passes over more than half of the last division and hence the required figure is greater than 5. Now if its distance appears to be nearly $\frac{3}{4}$ of the whole distance we might estimate it to be 7, but in this case it seems to fall short of this so we put down a 6, and hence the whole length is estimated to be 2·36″.

With a little practice this second figure after the decimal is obtained with great accuracy in measurements and the student should make serious efforts to get into the habit of estimating it.

Exercises 1a.

For the following exercises the student requires a rule divided on one edge into millimetres and centimetres and on the other edge into inches and tenths of an inch. The other side of the rule should be divided into inches, $\frac{1}{8}$ and $\frac{1}{16}$.

1. Draw the following lines; each line must be set out separately, the lengths being:—0·5″, 1·5″, 1·55″, 0·15″, 1·15″, 3·15″ 0·75″, 0·25″, and 0·712″.

2. Rule a line 5″ long and upon this line mark off consecutively the following distances starting from the left hand end. 0·5″, 1·25″, 0·4″, 0·35″, 0·2″, 1·51″, and 0·5″. Measure the total distance with the rule, estimating the final length to $\frac{1}{100}$″.

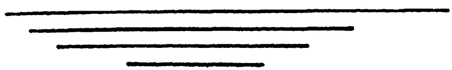

Fig. 4.

3. Estimate the length of the lines given in Fig. 4 to the nearest $\frac{1}{100}$ centimetre.

4. Rule a line 6 inches long and set out from the left hand end lengths of:—2·15″, 0·85″, 1·43″, 0·35″, 0·23″ following each other. Measure the total length and the remainder from the six inches and write the answers down to the nearest $\frac{1}{100}$″.

5. Draw lines 1·6, 4·8, 7·95, 12·7, 15·9, 19·1, 22·2, millimetres long and find by the aid of the ruler their nearest equivalents in sixteenths of an inch.

6. Draw lines 2·1, 5·1, 2·54, 10·16, 11·43, centimetres long and with the rule find their equivalent length in inches estimating the result to the nearest $\frac{1}{100}''$.

Decimals in the Workshop.—Enough has been said to show how suitable is the decimal system for all classes of work. In the early days of engineering the inch was subdivided into eighths, sixteenths, etc., but these are rapidly giving way to the decimal system now that more accurate work is in demand.

The following words are taken from a letter written by James Watt (the pioneer of the steam engine) about the year 1776:—"My dear friend, congratulate me, I have succeeded in making a cylinder nowhere more than a $\frac{1}{4}$ of an inch out of truth." In modern tool shops where accurate work is essential micrometers are used which will make measurements correct to $\frac{1}{1000}''$ (0·001″) and in some cases to $\frac{1}{10000}''$ (0·0001″). The more general use of the decimal system is a natural consequence of this advance towards accurate work, and the importance of the system for the student of any branch of engineering cannot be overstated. The general methods for dealing with quantities containing decimals will now be given.

Addition.

Ex. 2. *Add together:*—

12, 0·04, 3·7, 0·428, and 6·634.

In setting down an addition of whole numbers, we are careful to keep all the units' digits in one column and all the tens' digits in another and so forth. A similar care must be exercised here:—

$$\begin{array}{r@{\cdot}l}
12 & \\
0 & 04 \\
3 & 7 \\
0 & 428 \\
6 & 634 \\
\hline
22 & 802 \quad \text{Sum}
\end{array}$$

The operation is exactly the same as in an addition not containing decimals. The column on the right of the decimal point gave a total of 18, this is $\frac{18}{10} = 1\frac{8}{10} = 1{\cdot}8$. The 8 is therefore put down in this column and the 1 carried foward to the column containing the units' digits.

It is sometimes desirable to express a quantity as correct to one figure after the decimal point. The foregoing result would then be written **22·8.** If, however, we are asked to express it as the nearest whole number we should write it as **23,** since the first figure after the decimal is greater than 5. It is clear that in writing 23 we are only 0·2 in error, while 22 would have been 0·8 in error.

Exercises 1b.

1. Add together the following numbers :—

3·04
0·08
0·12
1·16

and if these numbers represent inches or parts of inches, set out their sum on a straight line and with the aid of the ruler find their nearest equivalent in millimetres.

2. Add together the following numbers and give the answer correct to the first figure after the decimal point :—

0·041 inches
1·094 ,,
0·875 ,,
1·250 ,,

Find with the aid of the ruler their nearest equivalent in inches and sixteenths.

3. Add together the following lengths in millimetres :—

19·1
22·2
49·2
41·3
11·6

Prove your answer graphically and find their nearest equivalent in inches and tenths.

4. Sum the following figures :—

$$\begin{array}{r} 12\cdot001 \\ 15\cdot008 \\ 20\cdot012 \\ 1\cdot018 \\ 3\cdot937 \\ \hline \end{array}$$

also give the result to the nearest whole number. If the numbers represent inches, find from the ruler the millimetre equivalent to the decimal part of the answer.

5. Add together the following :—

0·917	feet
0·989	„
0·976	„
0·899	„
0·524	„

Also write down the answer correct to one figure after the decimal point.

6. Add together :—

78·54	square inches
80·516	„
61·862	„
342·25	„
646·36	„

Give the answer correct to two figures after the decimal point.

Subtraction.

Ex. 3. *Subtract* 27·543 *from* 31·07.

Setting it down in the ordinary way and subtracting we get :—

$$\begin{array}{lr} & 31\cdot07 \\ & \underline{27\cdot543} \\ \text{Difference} & 3\cdot527 \end{array}$$

Note that the 3 in the lower line is treated as it would be if it stood under a 0. Also it is important to observe that the continuity of treatment is not broken when the decimal point is reached.

Ex. 4. (6·631 — 1·25) — (1·472 + 0·06)

Here the + sign indicates addition while the – sign instructs us to subtract. The quantities bracketed together must be treated first.

First bracket	6·631
	1·25
Difference	5·381
Second bracket	1·472
	0·06
Sum	1·532
Result of the first bracket	5·381
Result of the second bracket	1·532
Difference	3·849

Result, correct to three figures after the decimal, 3·849. This might be called the result to "four significant figures" or "four-figure accuracy." The result to three-figure accuracy would be 3·85.

Exercises 1c.

1. Add together

0·006
1·039
0·82

and subtract your answer from the sum of the following

3·937
3·858

Represent by lines the values of each of the sums of the above quantities correct to the first decimal place only.

2. Add together

0·57
10·52
151·605

and subtract the result from the sum of the following numbers, expressing the result to the nearest whole number.

1156·3
105·62
0·67

3. Subtract the following and give the answers correct to the first decimal place:—

116·504	0·580	19.719
105·6995	0·401	8.0801
99·865	9105·682	0·981
91·104	8134·001	0·106

4. Taking plus quantities as those measured to the right and minus quantities as those measured to the left, find the value of the following lengths combined: $+5''$, $-0·62''$, $+0·31''$, $-0·42''$, $-1·91''$, $-1·2''$. Prove your answer arithmetically and also find the value in centimetres from your ruler.

5. Subtract the sum of
12·699 millimetres
11·785 ,,
9·271 ,,

from the sum of
4·191 cm.
5·892 ,,
7·213 ,,

Give the result to the nearest millimetre.

Prove graphically that your answer is correct and find from your ruler the value of the answer in inches and tenths.

6. Subtract the sum of
171·961
501·615
170·310
0·625
0·715

from the sum of
5061·710
1·052
785·147
356·256

Express the result to the nearest whole number.

Multiplication (*ordinary method*).

Ex. 5. *Multiply* 15·362 *by* 3·21.

$$
\begin{array}{r}
15{\cdot}362 \\
3{\cdot}21 \\
\hline
15362 \\
30724 \\
46086 \\
\hline
49{\cdot}31202
\end{array}
$$

The *decimal point* is not placed until the product is completed and the rule is :—

Count the total number of figures after the decimal point in both numbers whose product is required, then mark off this number from the right hand end of the result.

Whether the student uses this or any other method for placing the decimal point he should *never* fail to apply the following common sense check. The above example obtains the product of two quantities, one rather larger than 15 and the other one larger than 3. It is perfectly obvious to anyone that the product should be something over 45, and this approximate value is sufficiently near the true result to prevent our making a foolish mistake in placing the decimal point.

In practical work it is highly probable that the above product would only be required to three-figure accuracy, in which case about half the labour of the calculation was wasted. The result is now obtained to three-figure accuracy by the

Contracted Method of Multiplication.

Ex. 6. *Multiply* 15·362 *by* 3·21 *correct to three significant figures.*

Since we require three-figure accuracy we must aim at keeping four figures in the result. In order to do this we need only retain four figures in the upper line (called the "multiplicand"). Hence in setting down the problem the fifth figure (a 2) is crossed off, thus :—

$$
\begin{array}{r}
15{\cdot}36\cancel{2} \\
3{\cdot}21 \\
\hline
46{\cdot}081
\end{array}
$$

The multiplicand is then multiplied by the *left* hand figure in the multiplier (in this case a 3) and the decimal point placed by our common sense rule. A vertical line may then be drawn as shown. Next the digits in the multiplier are numbered (1, 2, 3, etc.) from the left hand end, and in the multiplicand the digits are numbered from the right hand end, thus:—

	4 3 2 1
	15·36~~2~~
	1 2 3
	3·21
Line 1.	46·08
Line 2.	3·06
Line 3.	·15
Sum to three significant figures,	49·3

Line 1 has already been explained. Line 2 is the upper line multiplied by 2, and since this is No. 2 digit of the multiplier we start multiplying at No. 2 digit in the upper line (viz. 3). Line 3 is the upper line multiplied by 1 (No. 3 digit) and starting at the 5 (No. 3 digit in the upper line). Note that the sum is 49·29, but as only three-figure accuracy is required it is written down 49.3.

When the left hand figure in the multiplicand is sufficiently large, it will be found that three figures only need be retained in order to give four figures in the first line of the multiplication. Whether this is the case or not may be found by inspection. Cases may arise in which instead of there being too *many* figures in the multiplicand there are too *few*, noughts may then be added to make up the required number.

Ex. 7. *The length of the circumference of a circle is* 3·1416 *times the length of the diameter. Find the length of the circumference of a circle whose diameter is* 6·125″. *(Three-figure accuracy is required.)*

3 2 1	
6·12~~5~~	
1 2 3 4 5	
3·1416	
18·36	
·61	
·24	
19·2	*Result*, 19·2 inches.

Before leaving the subject of multiplication, it should be noted that multiplying any decimal quantity by 10 or any "power of

10" (*i.e.* 100, 1,000, etc.) merely has the effect of moving the decimal point towards the right. Thus

$$15{\cdot}736 \times 10 = 157{\cdot}36$$
$$15{\cdot}736 \times 100 = 1573{\cdot}6$$

Similarly

$$15{\cdot}736 \div 10 = 1{\cdot}5736$$
$$15{\cdot}736 \div 100 = 0{\cdot}15736$$
$$15{\cdot}736 \div 1000 = 0{\cdot}015736.$$

Exercises 1d.

1. Multiply the following numbers together by the contracted method giving the answers correct to three significant figures only:—

151·6 by 15·9, 1·06 by 5·99, 3·1416 by 3·1416,
0·0965 by 61·9, 17·256 by 0 981.

2. Multiply together the following numbers and give the answers correct to the first figure after the decimal point:—

7·956 by 55·99, 235·6 by 0·0591, 0·5621 by 155·732,
0·995 by 0·995, 109 by 0·109, 56·73 by 6·731.

3. Multiply the following numbers together:—

0·464 by 0·464, 0·324 by 0·342, 0·212 by 0·212.

In each of the above examples multiply the answer by 0·7854 and give the result correct to the third figure after the decimal point.

4. If 0·39370 is the factor by which a measurement in centimetres must be multiplied by to convert it to inches, convert the following measurements in centimetres to inches and the nearest $\frac{1}{100}''$:— 25·394, 2·5409, 199·56, 0·962.

5. If a metre is equal to 39·370113 inches, find the value in inches of:—12·65, 2·79, 6·7, and 0·95 metres. Use the contracted method and give the values correct to the first figure after the decimal point.

6. Multiply together the following, but in each case try to write the approximate answer down first and then calculate the result by the contracted method to the first figure after the decimal point.

$6{\cdot}456 \times 6{\cdot}4516$, $8{\cdot}064 \times 38{\cdot}70$, $0{\cdot}155 \times 10{\cdot}76$, $96{\cdot}87 \times 1.395$.

Division.

Ex. 8. *Divide* 58·742 *by* 16·27.

It is a good plan always to arrange to have one digit only before the decimal point in the divisor. This can always be accomplished by multiplying or dividing by some power of 10, and of course the dividend must be multiplied or divided by the same quantity. Dividing both quantities by 10 in the above example we have :—

1·627)5·8742(3·61
4881
9932
9762
1700
1627
73

The position of the decimal point should present no difficulty when we consider that we are dividing a quantity (1·627) between 1 and 2 into another (5·87, etc.) between 5 and 6.

Note that when there were no more figures in the dividend to "bring down" a 0 was added to the remainder in each case. As there is nothing to prevent this process being carried on indefinitely, care must be exercised to confine the result to the number of significant figures required.

Contracted Division.

Ex. 9. *Divide* 15·25 *by* 3·1416 *correct to three significant figures.*

It is usual to retain only four figures in the divisor when three-figure accuracy is required. The fifth figure (a 6) is therefore discarded. When the discarded figure is larger than 5 we add 1 on to the last retained figure. The problem is now set down thus :—

$3\cdot1\cancel{4}\cancel{2})15\cdot250(4\cdot86$ nearly.
12·568
2·682
2·512
170

Result to three-figure accuracy, 4·86.

It will be observed that instead of adding a 0 to each remainder, the last digit in the divisor is discarded.

Significant Figures.—The student must exercise great care in deciding how many significant figures should be retained in a result.

Here is an example:—A workman measures the diameter of a cylinder and finds it to be $7\frac{1}{8}''$. This length is required in centimetres, and as one inch equals 2·54 centimetres, and $\frac{1}{8}''$ equals 0·125″, the result will be obtained by multiplying 7·125″ by 2·54. Multiplying this by the ordinary method the result is 18·09750 centimetres. If all these figures are to have a real significance the measurement must be correct to $\frac{1}{100000}$ part of a centimetre. Now the workman made the original measurement to the nearest $\frac{1}{8}''$ and he therefore might have been at least a millimetre in error. Therefore it is more correct to express the result as 18·1 centimetres.

It should here be noted that if a calculation involves three or four steps and contains no addition or subtraction, then to get three-figure accuracy we must work throughout to five significant figures.

Exercises 1e.

1. Divide and express as a decimal:—

1 by 2, 1 by 4, 1 by 8, 1 by 16, 1 by 100.

If the results represent lengths in inches, draw lines to represent them.

2. Find the value in decimals of:—

1 divided by 32, 1 by 25, 1 by 40, 1 by (40 × 35), 1 by 64. Explain the results graphically where possible.

3. Divide:—

25·4 by 5·392, 3·14 by 0·094, 75·24 by 0·874, 88·16 by 111·9.

Use the contracted method and give the answers correct to the first figure after the decimal point.

4. Find the value of the following:—

$\frac{25}{0{\cdot}1}$, $\frac{16}{0{\cdot}125}$, $\frac{169}{0{\cdot}37}$, $\frac{0{\cdot}1}{25}$, $\frac{0{\cdot}125}{16}$, $\frac{0{\cdot}37}{169}$.

Explain from the results the effect of a number less than unity in the denominator.

5. Find the value of the following to three-figure accuracy:—

$\frac{51{\cdot}123}{0{\cdot}785}$, $\frac{0{\cdot}375}{0{\cdot}785}$, 0·785 divided by 78·5.

Try to write an approximate answer down in each case before commencing the calculation.

6. Find the value of the following:—

$$\frac{22}{7},\quad \frac{22}{28},\quad \frac{1\cdot56}{0\cdot064},\quad \frac{969\cdot6}{18},\quad 0\cdot624 \text{ divided by } 74\cdot6.$$

Express the results correct to three-figure accuracy.

Exercises 1f.

1. Find the value of the sum of the following lengths with the aid of the ruler:—0·75″, $\frac{15}{16}''$, $\frac{1}{8}''$, $\frac{1}{16}''$, 0·625″, and 0·375″, check the results by converting $\frac{15}{16}''$, $\frac{1}{8}''$, and $\frac{1}{16}''$ into decimals and then adding together.

2. Add together 2·56″, 0·78″, 1·67″, and subtract the sum from the sum of 20·8, 3·111, and 5·191 centimetres. (An inch is equal to 2·54 centimetres.) Express the answer in both centimetres and inches.

3. Find the remainder (with the ruler and arithmetically) of:—

$$2\cdot015'', \quad + 3.54'', \quad - 1\cdot078'', \quad - 0\cdot235''.$$

If dividing by 0·3937, convert inches into centimetres, find the value in centimetres and prove the answer with the ruler.

4. To convert miles into kilometres the miles must be either multiplied by 1·60934 or divided by 0·62137. Prove that both figures give the same results in the following cases:—29·75, 10·65, 212·55, and 0·75 miles by converting them into kilometres.

5. Find the value of the following:—

$$\left(\frac{176\cdot5}{0\cdot99}\right) \times \left(\frac{56\cdot13}{110}\right) - \left(\frac{1\cdot9}{16}\right), \text{ and also } \frac{78 \times 13 \times 23\cdot24 \times 300}{33000 \times 12}.$$

6. Find the value of the following to the first figure after the decimal point:—

$$(0\cdot78 \times 56\cdot1) + (76\cdot2 \times 0\cdot86) - (5\cdot16 \times 1\cdot16).$$

CHAPTER II.

VULGAR FRACTIONS.

Numerator and Denominator.—The fractions most commonly met with in practical measurement are :—$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{32}$, and $\frac{1}{64}$, and multiples of these. Let us think of a fraction such as $\frac{5}{8}$. If this relate to an inch it means that a length of 1 inch has been divided into 8 equal parts, and the fraction indicates 5 of these parts. This idea is shown in Fig. 5.

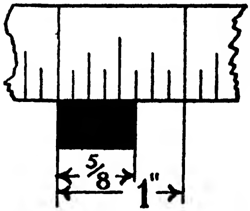

Fig. 5.

Another way of regarding it is that it indicates 5 divided by 8, in which case, if an inch is the unit to which it refers, it is one eighth part of five inches. The student should now draw a line 5″ long and divide it into 8 equal parts by the method described in Chapter I., and verify for himself that each of these divisions is equal to $\frac{5}{8}''$ as measured on his rule.

In such a fraction the 8 is called the **denominator** and the 5 the **numerator.**

Returning to Fig. 5, it can be seen that if each of the divisions were subdivided into two equal parts, the inch would be divided into 16, and what is now $\frac{5}{8}$ would be $\frac{10}{16}$. In other words $\frac{5}{8}=\frac{10}{16}$. In this way it may be seen that a fraction remains unaltered in value when the numerator and denominator are *both* multiplied by the same quantity. For example $\frac{7}{16} = \frac{7 \times 3}{16 \times 3}$.

Addition and Subtraction of Fractions.

Ex. 10. *Find the value of* :—

$$\tfrac{1}{2} + \tfrac{3}{4} - \tfrac{7}{8} + \tfrac{3}{16}.$$

The first thing to do is to find a *common denominator*, i.e. a number which is a multiple of the denominators of all the fractions. This is sometimes called the L.C.M., which means "Least Common Multiple." It is

most convenient to use the "least" (*i.e.* smallest) number which will answer the purpose, but it is not absolutely *necessary* to do so. In the example given above 16 is the L.C.M. Now to express $\frac{1}{2}$ in 16ths we must multiply by $\frac{8}{8}$ $\left(\text{because } \frac{16}{2} = 8\right)$ and in the same way $\frac{3}{4}$ must be multiplied by $\frac{4}{4}$ $\left(\text{because } \frac{16}{4} = 4\right)$. Thus the expression becomes :—

$$\left(\frac{1 \times 8}{2 \times 8}\right) + \left(\frac{3 \times 4}{4 \times 4}\right) - \left(\frac{7 \times 2}{8 \times 2}\right) + \frac{3}{16}.$$

$$= \frac{8 + 12 - 14 + 3}{16}$$

$$= \frac{23 - 14}{16} = \frac{9}{16}.$$

The student should now set out along a straight line, distances corresponding to these quantities, taking 1″ as the unit.

Ex. 11. *Find the value of* :—

$$3\tfrac{7}{8} + 2\tfrac{1}{4} - 3\tfrac{5}{32}.$$

Taking the whole numbers first :—

$$3 + 2 - 3 = 2.$$

Now the fractions (the L.C.M. of the denominators is 32) :—

$$\left(\frac{7 \times 4}{8 \times 4}\right) + \left(\frac{1 \times 8}{4 \times 8}\right) - \frac{5}{32}$$

$$= \frac{28 + 8 - 5}{32}$$

$$= \frac{31}{32}$$

Result, $2\frac{31}{32}$.

This result should be verified by stepping out the distances along a line as in Example 10. Use 1″ as the unit.

Exercises 2a.

In each of the following exercises the student should first find the value of the expression by the arithmetical method just shown, and then verify the result by stepping out the distances along a straight line, putting in the dimensions; + quantities should be measured from left to right and − quantities in the opposite direction.

1. $\frac{1}{8}'' + \frac{3}{4}'' - \frac{5}{16}'' + \frac{1}{4}$ **2.** $\frac{27}{32}'' + \frac{15}{16}'' - \frac{7}{8}''$

3. $\frac{7}{12} - \frac{1}{4} + \frac{5}{8}$ **4.** $2\frac{3}{16} - 1\frac{1}{4} + 1\frac{5}{8}$

5. $3\frac{5}{32} + 2\frac{1}{8} - 2\frac{7}{16}$ **6.** $4\frac{1}{4} - 3\frac{13}{64} + 2\frac{15}{32}$

Multiplication of Fractions.—If a man walk at the rate of 3 miles per hour for 4 hours, how many miles will he have travelled? The answer to this problem is obviously 12 miles, and it is obtained by multiplying the velocity (3 miles per hour) by the time (4 hours). We may therefore write

$$\text{Distance} = \text{Velocity} \times \text{Time}.$$

Let us think of another example. How far will a man swim in $\frac{2}{7}$ of an hour at the rate of $\frac{3}{5}$ of a mile per hour? It is easily seen that he will swim $\frac{2}{7}$ of $\frac{3}{5}$ of a mile. Hence it follows that $\frac{2}{7}$ of $\frac{3}{5}$ means $\frac{2}{7} \times \frac{3}{5}$.

Regarding $\frac{1}{5}$ as the unit and dividing it into 7 equal parts, we get, 1 fifth = 7 thirty-fifths.

$\therefore$ 3 fifths = 21 thirty-fifths. Therefore 1 seventh of 3 fifths = 3 thirty-fifths and 2 sevenths of 3 fifths = 6 thirty-fifths.

We are now in a position to state the

Rule for the Multiplication of Fractions.—*Multiply the numerators together to get the numerator of the product, and multiply the denominators together to get the denominator of the product.*

The fraction should afterwards be cancelled down to its simplest form.

Division of Fractions.—When one fraction is to be divided by another, we are really asked to find a fraction which, when multiplied by the divisor, gives the dividend. From this it follows that $\frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \times \frac{7}{2}$ because $(\frac{3}{5} \times \frac{7}{2})\, \frac{2}{7} = \frac{3}{5}$.

i.e. Answer $\times$ divisor = dividend.

Rule for the Division of Fractions.—*Invert the Divisor Fraction and Multiply.*

A whole number may be regarded as a fraction in which it forms the numerator, the denominator being 1. Thus a quantity such as $4\frac{3}{16}$ (which is called a "mixed number") may be regarded

as $\frac{4}{1} + \frac{3}{16}$, and to express this with a common denominator (*i.e.* in 16ths), the 4 must be multiplied by 16 and the expression becomes $\frac{64+3}{16} = \frac{67}{16}$. This is sometimes called an "improper fraction," and it is convenient to convert all mixed numbers into such when they enter into multiplication or division operations.

Ex. 12. *Simplify*:—

$$2\tfrac{1}{4} \times \tfrac{3}{8} \times 1\tfrac{1}{2} \div \tfrac{3}{16}.$$

Removing the mixed numbers... .. .. $\frac{9}{4} \times \frac{3}{8} \times \frac{3}{2} \div \frac{3}{16}$.

Inverting the divisor fraction .... . $\frac{9}{4} \times \frac{3}{8} \times \frac{3}{2} \times \frac{16}{3}$.

Now we may cancel a common factor in a denominator and *any* other numerator. Thus the 8 in the denominator of the second fraction cancels with an 8 which is one of the factors of the 16 which forms the numerator of the fourth fraction.

That is $\frac{9}{4} \times \frac{3}{\cancel{8}} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{\overset{\cancel{2}}{\cancel{16}}}{\cancel{3}}$

$= \frac{27}{4}$

The two 3's which are "cancelled out" really leave 1's in their places, but as these will leave the value unaffected when we multiply out, they are often omitted. The result $\frac{27}{4}$ is an improper fraction. To convert it into a mixed number, we divide the 27 by 4 (result 6) and the remainder is placed over the 4. Thus: $\frac{27}{4} = 6\frac{3}{4}$.

Exercises 2b.

Find the value of :—

1. $\frac{3}{4} \times 2\frac{7}{8} \times \frac{2}{3} \times \frac{1}{2}$.

2. $2\frac{5}{8} \times 1\frac{3}{7} \times \frac{3}{5}$.

3. $3\frac{3}{16} \times \frac{7}{16} \div \frac{7}{8}$.

4. $4\frac{3}{8} \div \frac{5}{16} \times 2\frac{2}{3} \times \frac{3}{8}$.

5. $3\frac{3}{4} \times 1\frac{3}{32} \div 1\frac{7}{8}$.

6. $2\frac{2}{64} \div \frac{17}{32} \times 1\frac{3}{4}$.

To Convert a Vulgar Fraction into a Decimal.—The use of decimals is such an aid to calculation that it often becomes necessary to convert a fraction into an equivalent decimal.

Ex. 13. *Convert $\frac{3}{8}$ into a decimal*:—

This fraction indicates $3 \div 8$. Now the 3 may be regarded as 3·00000 . . . Dividing by 8 we get

8)3·0000
·375.

Note the ·375 is an *exact* equivalent of $\frac{3}{8}$, as the 3 multiplied by some power of 10 happens to be an exact multiple of 8, but in many cases no such relationship exists, and then division must be carried on until the required number of significant figures is obtained.

Ex. 14. *Convert* $\frac{7}{12}$ *into a decimal* :—

(Four-figure accuracy is required.) Proceeding as before :—

$$12\overline{)7{\cdot}0000}$$
$$\underline{{\cdot}5833}.$$

The student should make for himself a table giving the decimal equivalents of the commonly used fractions of an inch, viz. :—$\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$.

Exercises 2c.

The decimal equivalents of the following fractions should be found by the method shown in Example 14. Straight lines of these lengths should then be drawn as accurately as possible by means of the rule, using the inch scale graduated in eighths, &c., and then measured by means of the inch scale graduated in tenths. This result should be compared with the calculated value.

1. $1\frac{3}{4}$. **2.** $2\frac{5}{8}$. **3.** $2\frac{5}{12}$. **4.** $1\frac{7}{16}$. **5.** $3\frac{13}{32}$. **6.** $2\frac{25}{64}$.

To Convert a Decimal into a Vulgar Fraction.—This is not a very common operation in practical calculations, nevertheless the student should know how to proceed.

Ex. 15. *Convert* 0·1875 *into a fraction.*

Now this is equal to $\frac{1}{10} + \frac{8}{100} + \frac{7}{1000} + \frac{5}{10000}$

$$= \frac{1000 + 800 + 70 + 5}{10000}$$
$$= \frac{1875}{10000}.$$

From this result we obtain the

Rule.—*Place the decimal quantity in the Numerator and use for the Denominator a* 1 *followed by as many noughts as there are figures in the decimal.*

The result must now be cancelled down to its simplest form :—

$$\frac{1875}{10000}$$

Cancelling by 25 .... $= \frac{75}{400}$

Cancelling by 25 again .... $= \frac{3}{16}$

Exercises 2d.

Convert the following decimals into fractions, expressing the latter in their simplest form :—

1. 0·25. **2.** 1·125. **3.** 2·375

4. 0·0625. **5.** 1·4375 **6.** 0·28125.

Exercises 2e.

1. Multiply $3\frac{15}{16}$ by $\frac{4}{9}$ and convert the answer to a decimal. Verify the result by drawing a straight line and marking off a length of $3\frac{15}{16}''$. Divide this length into 9 equal parts and measure the total length of 4 of them.

2. Multiply $7\frac{7}{10}$ by $\frac{3}{7}$ and verify it as in No. 1, but using 1cm. as the unit of length.

3. Add together $1\frac{1}{4}$ and $2\frac{5}{16}$. Multiply the sum by $\frac{2}{3}$ and divide the product by $\frac{3}{8}$.

N.B.—This problem might have been set thus :—

$$\frac{\frac{2}{3}(1\frac{1}{4} + 2\frac{5}{16})}{\frac{3}{8}}.$$

The $\frac{2}{3}$ placed immediately in front of the bracket indicates that it should be multiplied by the whole quantity inside the bracket.

4. Simplify :— $$\frac{\frac{8}{11}(2\frac{5}{8} + 1\frac{5}{32})}{3\frac{2}{3}}$$

5. A rod is $36\frac{3}{4}''$ long. A length of 21″ is cut off. What fraction of the whole length is this ?

(If you cannot see how to start this, think of a simple case :— Suppose the rod were 8″ long and the cut off portion had been 4″ ; this is obviously $\frac{1}{2}$ of the whole length. Ask yourself why ? and apply the same method to the problem set.)

6. A rod is $8\frac{3}{4}''$ long; $\frac{3}{7}$ of its whole length is cut off; what length remains ?

CHAPTER III.

POWERS AND ROOTS.

Squares.—We are accustomed to speak of " Square Measure " when we are dealing with the measurement of area, and a unit frequently used is a "*square* foot"; by this we mean the area of a square whose edge is 1 foot long.

Now if we consider a square whose edge is 3 feet long, we know that its area is 9 square feet, for if the square were divided up into square feet there would be three rows of squares with three in each row, as shown in Fig. 6. In other words the number of square feet in this square is (3×3). It is convenient to write this as 3^2, which is read "3 squared," the little 2 being called the **index**. In this way we see that 6^2 equals $6 \times 6 = 36$ and $9^2 = 81$ and so forth, each quantity representing the area of a square.

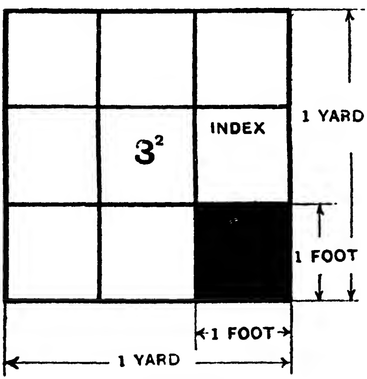

Fig. 6.

Cubes.—In just the same way 5^3, which is read " 5 cubed," $= 5 \times 5 \times 5 = 125$, and this represents the volume of a cube whose edge is 5 units long. Following the same idea 3^4 (3 to the fourth) $= 3 \times 3 \times 3 \times 3 = 81$, and $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, but no geometrical meaning can be given to such expressions.

We can now see how it is that since 1 foot $= 12''$, 1 sq. ft. $= 12 \times 12 = 144$ square inches, and 1 cubic ft. $= 12 \times 12 \times 12 = 1728$ cubic inches. Similarly since 1 metre $= 100$ centimetres (cm.), 1 sq. metre $= 100^2 = 10{,}000$ sq. cm. and 1 cubic metre $= 100^3 = 1{,}000{,}000$ cubic cm. It is the custom of some people to write ft.^{2} for square foot and cm.^{3} for cubic centimetre and so forth.

Powers.—Instead of speaking of 9 as the "square of 3," it is sometimes spoken of as 3 raised to the **power** of 2. Similarly 4 raised to the power of 3 is $4^3 = 64$.

Ex. 16. *A square has an edge of 4ft. 3in., find its area to the nearest square foot.*

The area of this square might be expressed by (4ft. 3in.)^2, but before such an expression can be squared it must be expressed in one unit only (*i.e.* either feet or inches). The student should note very carefully that it is wrong to square the 4 ft. ($= 16$ ft.^{2}), and the 3 in. ($= 9$ in.^{2}) and add the results together He must make up his mind which will be the more convenient, (1) to work in inches, or (2) to work in feet and a decimal. The latter will be better in this case, as 3 in. $= \frac{3}{12}$ ft. $= \frac{1}{4}$ ft. $= 0{\cdot}25$ ft.

The area of the square is therefore $(4{\cdot}25)^2$ sq. ft. Contracting so as to have only one figure after the decimal we have:—

$$\begin{array}{r}
\scriptstyle 2\ 1 \\
4{\cdot}2\cancel{5} \\
\scriptstyle 1\ 2 \\
4{\cdot}2\cancel{5} \\
\hline
16{\cdot}8 \\
8 \\
\hline
17{\cdot}6
\end{array}$$

Result to nearest square foot, 18 ft.^{2}.

The student should apply a rough check to this result. The side of the square had a length between 4 and 5 feet, and its area should therefore lie between 4^2 (16) and 5^2 (25) sq. ft.

Ex. 17. *Find the volume of a cube whose edge is 7 ft. $1\frac{1}{2}$ in. long. (Result to the nearest cubic foot.)*

Converting the inches to decimal of a foot :—

$$1\tfrac{1}{2}\text{ in.} = \frac{1\frac{1}{2}}{12}\text{ ft.} = \frac{3}{24}\text{ ft.} = \frac{1}{8}\text{ ft.} = 0{\cdot}125\text{ ft.}$$

The volume of the cube = $(7{\cdot}125)^3$ cub. ft. That is, we have to multiply 7·125 by 7·125, and the product obtained is multiplied by 7·125. Now we have to contract our working so as to obtain one figure after the decimal in the final product. Hence the first product must contain more than this, otherwise the final result will be affected. Let us therefore aim at getting two figures after the decimal in the first product.

$$
\begin{array}{rr|ccr|l}
 & {\scriptstyle 4\,3\,2\,1} & & & {\scriptstyle 4\,3\,2\,1} & \\
 & 7{\cdot}125 & & & 50{\cdot}76 & \\
 & {\scriptstyle 1\,\,2\,3\,4} & & & {\scriptstyle 1\,\,2\,3\,4} & \\
 & 7{\cdot}125 & & & 7{\cdot}125 & \\
\cline{2-2}\cline{5-5}
 & 49{\cdot}875 & & & 355{\cdot}32 & \\
 & {\cdot}712 & & & 5{\cdot}07 & \\
 & {\cdot}142 & & & 1{\cdot}00 & \\
 & {\cdot}035 & & & {\cdot}25 & \\
\cline{2-2}\cline{5-5}
(7{\cdot}125)^2 \ldots\ldots\ldots & 50{\cdot}76 & & & 361{\cdot}6 & \ldots\ldots\ldots\ (7{\cdot}125)^3
\end{array}
$$

Result (to the nearest cubic foot), 362 ft.^{3}.

Rough check :—The volume will be rather more than 7^3, and 7×7 is roughly 50, and $7 \times 50 = 350$.

The student should very carefully note the necessity of keeping *four* significant figures in *both* multiplications in order to get the *three* significant figures of the result accurate.

Exercises 3a.

1. Given that 1 inch = 2·54 cm., find the number of square cm. in 1 sq. in. (Three-figure accuracy.)

2. Given that 1 metre = 39·37 in., find the number of sq. in. in 1 sq. metre. (Four-figure accuracy.)

3. Find the area of a square whose edge is 3 ft. $7\frac{1}{2}$ in. long. (To the nearest sq. foot.)

4. Find the number of cubic centimetres in 1 cubic in. (Three-figure accuracy, use datum given in No. 1.)

5. Find the number of cubic *feet* in 1 cubic metre. (Three-figure accuracy, use datum given in No. 2.)

6. Find the volume of a cube whose edge is 15·63 cm. long. (To the nearest cubic cm.)

Roots.—A "root" of a number is that quantity which *multiplied by itself a given number of times produces the number.* A root is indicated by the symbol $\sqrt{}$ which is sometimes called a "radical sign." Thus $\sqrt[2]{9}$ is read "the square root of 9," and we must look for a quantity which, multiplied by itself, gives 9, viz. 3. The square root sign is so frequently used that the 2 is usually omitted. Again $\sqrt{8}$ indicates "the cube root of 8" which is 2, because $2 \times 2 \times 2 = 8$, and 3 is the 4th root of 81 ($\sqrt{81}$) because $3 \times 3 \times 3 \times 3 = 81$.

Square Root.

Ex. 18. *Find the square root of* 221841·0.

The number must be set down and the digits marked off in pairs, starting from the decimal point to the left for unit digits, and to the right for numbers less than unity. In the following example the lines and columns are marked and the operation explained below :—

	A.	B.	C.
1st Line	4	22 | 18 | 41	471
2nd „		16	
3rd „	87	618	
4th „		609	
5th „	941	941	
6th „		941	

The number having had its digits marked off in pairs is placed in Column B. Find, by trial, the largest number the square of which is less than the marked off quantity on the left (in this case 22). 4 is the number required. Place it in Column C (which is reserved for the result) and also in Column A. Place its square (16) under the 22, subtract and bring down the next pair of digits. This brings us to the third line. In Column A double the 4 and bring it down to this line. Find by trial that the 8 divides into 61, 7 times. Place the 7 in Column C and also on the right hand side of the 8, making 87 in Column A. Now multiply 87 by 7 and put the result under the 618, subtract and bring down the next pair of digits. In Column A bring down the 87, with the last digit doubled (making 94) and put it in the fifth line. Repeat the process adopted in line 3 and the final result is given as 471.

In line 3 the 8 divides into 61 7 times, the 7 being placed beside the 8 and also in column C. Now it often happens that on being multiplied out too large a number is obtained in line 4,

in which case the new figure (in this case a 7) has to be replaced by a smaller one which is found by trial. This is illustrated in the following examples.

As a rough check we may regard the result as rather less than 500, the square of which is 250,000. This is sufficiently near the original number (221,841·0) to show that our result is at least a sensible one.

The quantity dealt with in this example happened to be a perfect square. When such is not the case, the root may be carried into a decimal, obtaining as many significant figures as may be required.

Ex. 19. *Find the square root of* 352.

This may be looked upon as 352·00. Marking off pairs we get :—

1	352·0000	18·76
	1	
28	252	
	224	
367	2800	
	2569	
3746	23100	
	22476	

Rough check, 18·76 is nearly 20 $(20)^2 = 400$. Original number 352.

Ex. 20. *Find the square root of* 2. (*Four-figure accuracy.*)

	2·000000	1·414
1	1	
24	1·00	
	96	
281	400	
	281	
2824	11900	
	11296	

If the student obtains approximate values for the $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ he will find them of considerable use. Thus

$$\sqrt{50} = \sqrt{25 \times 2} = \sqrt{5^2 \times 2} = 5\sqrt{2} = 5 \times 1·414 = 7·07$$

Rough check, $7^2 = 49$, which is nearly 50.

Again $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{3^2 \times 5} = 3\sqrt{5}$.

The extraction of a square root is used when we require the length of the edge of a square whose area is known. The method of extracting cube roots should be postponed until the student has proceeded a little farther in his mathematical work.

Exercises 3b.

1. Find the value of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$. (Four-figure accuracy.)

2. Using the values found in the above question find the values of $\sqrt{32}$, $\sqrt{75}$, $\sqrt{28}$.

3. Find the square root of 64009.

4. Find the square root of 527.

5. An acre = 4840 square yards. Find the length of the edge of a square field whose area is 1 acre. (Result to the nearest foot.)

6. A sheet of 4to paper is found to be 25·86 cm. long and 20·22 cm. wide Find to the nearest millimetre the length of the edge of a square of equal area.

CHAPTER IV.

USE OF SQUARED PAPER : GRAPHS.

Scales.—On looking at a drawing of a piece of machinery there will be seen the word "Scale," and under it some remark such as: "full size" or "2″ to the foot," or a scale may be placed upon it as shown in Fig. 7.

Fig. 7.

Again, a map of a district may be obtained on which a mile is represented by one inch. In these and similar cases we have a scale on which comparatively small lengths are made to represent longer ones.

Now, just as 1″ can represent a mile, so we might make a scale in which that (or any other) length might represent a weight such as 100 pounds, or a period of time, as one minute, or even a cost of something in £'s per ton.

In Fig. 8 we have what is called a **graph**. The vertical scale represents the cost of copper in £'s per ton. The horizontal scale is one of time, the positions marked being the market days of the first three weeks in a given month. The graph was obtained as follows:—On Monday in the first week copper sold at £76 19s. per ton, hence we mark a point on the chart vertically above the position on the horizontal scale indicating this day, and opposite the position on the cost scale representing £76 19s. Now 19s. = £$\frac{19}{20}$. Therefore, to find this point the distance between the points indicating £76 and £77 must be divided into 20 equal parts, and 19 of them marked off. This point being fixed we proceed to the next day and so forth.

When all the points are marked a line is drawn through them as illustrated. This line shows at a glance how the price of copper rose and fell during the month, better than any list of

figures could do. Moreover, the price on any particular day could easily be determined. Thus, if we require the price on Tuesday in the third week, we look at the graph vertically above the date and find that it is exactly opposite £74 per ton.

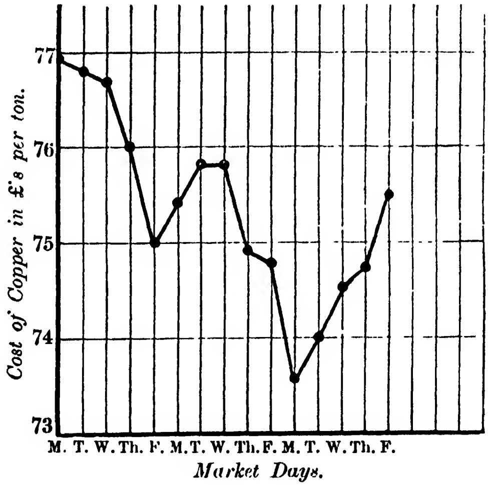

Fig. 8.

For the purpose of plotting "graphs" as this operation is called, it is convenient to have **squared paper**. This is paper ruled in squares with 1″ sides, each square being subdivided by fainter rulings, usually one-tenth of an inch apart. Fig. 9 shows a graph plotted on such paper.

The student should provide himself with some of this paper, and should plot the graphs described in this and subsequent chapters. He should never lose an opportunity of plotting a graph representing the result of any experiment he may perform.

Ex. 21. *A piece of "Delta Metal" wire about 8′ long was suspended from a beam and weights were hung on the free end. These weights (called "the load") caused the wire to stretch. The*

amount of stretching (called "the extension") was carefully measured for various loads and recorded thus:—

Load in Pounds.	Extension in Inches.
2	0·08
3	0·12
4	0·16
5	0·20
6	0·24

Plot a graph showing the relation between the load and the extension.

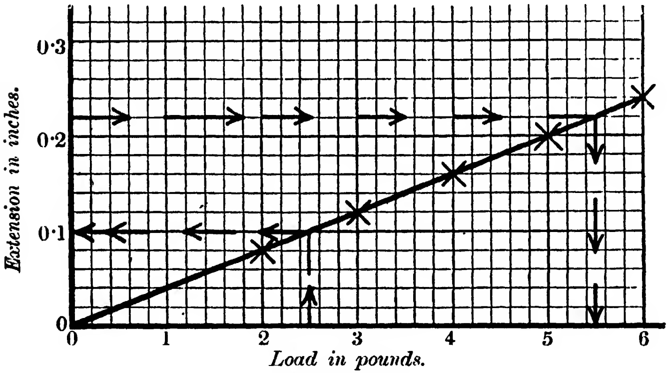

Fig. 9.

We must first decide upon our scales. It will be convenient to have the load scale horizontal, taking 1″ = 1 lb., and the extension scale vertical, taking 1″ = 0·1″. In the latter, each $\frac{1}{10}$″ of the scale is equivalent to 0·01″ of the extension, and consequently the second reading (0·12), for example, is two small divisions past the position mark 0·1″. Fig. 9 shows the scales and the graph drawn to half size. The "plotted" points are marked with a cross so that we may locate them after the graph is drawn through these points.

It should be here noted that the graph is a straight line. The significance of this fact will be dealt with in a subsequent chapter.

We may now use this graph to find what the extension would be for any load between 2 and 6 lb. For example:—Find the extension caused by a load of $2\frac{1}{2}$ lb. We must first locate this amount on the load scale. Now ten of these divisions into which each inch is divided represent 1 lb., so each small division represents 0·1 lb.; again $\frac{1}{2}$ lb. = ·5 lb., and this is represented by 5 divisions. Take a point, therefore, 5 divisions past the point representing 2 lb., and travel vertically upwards to the graph. It will be found that this point so obtained is opposite 0·1″ on the extension scale, which is the extension required.

This process is called **interpolation**, and it may be employed in precisely the same way for finding the load required to produce a given extension. Thus: to produce an extension of 0·22″ a load of $5\frac{1}{2}$ lb. will be required.

This process of interpolation is of considerable use in a number of common operations.

Graph for the Determination of Square Roots.

Ex. 22. *Plot a graph showing the relation between the numbers from* 1 *to* 10 *and their square roots.*

Extracting a few square roots by the methods described in the last chapter we may compile the following table:—

Number.	Square Root.
1	1
2	1·41
3	1·73
4	2
6	2·45
8	$2\sqrt{2} = 2{\cdot}82$
10	3·16

Plotting these values we obtain a graph similar to that shown in Fig. 10.

It will be noticed that the points marked for the graph shown in Fig. 10 are joined by a *curve*, while those obtained in Fig. 8 were joined by *straight lines*. There is this difference in the two

cases:—The price of copper was not regulated by any mathematical law, but jumped suddenly from one price to another in consequence of such accidental conditions as supply and demand. On the other hand, the square root of our numbers grows gradually with the increase of the number, and if we were

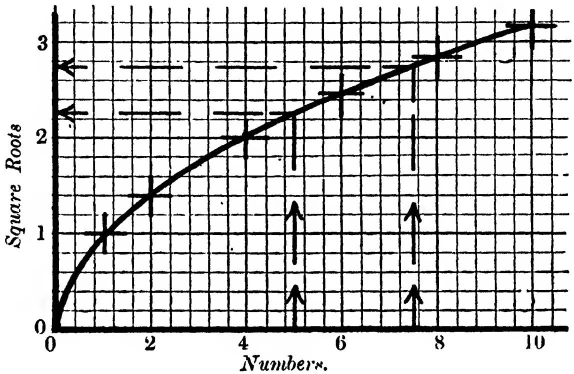

Fig. 10.

to calculate the square root of intermediate values such as 1·2, 1·4, etc., we should find that these new points would suggest the *curve* and prove conclusively that a succession of straight lines would be wrong.

From Fig. 10 the square root of any number from 1 to 10 might be obtained; for example, the dotted lines show that the square root of $5 = 2{\cdot}24$, and that the square root of $7{\cdot}5 = 2{\cdot}74$.

Graph for Converting Inches into Centimetres.

Ex. 23. *Given that* $1'' = 2{\cdot}54$ *cm., draw a graph for converting lengths between* $1''$ *and* $10''$ *into cm.*

If we could be sure that the graph is a straight line, we need only plot two points and then draw a straight line through them. With our present experience, however, we cannot say definitely, so we must make a test by calculating three points. If these fall in a straight line well and good, if not, more points must be plotted in order to get the correct curve.

TABLE OF VALUES.

Length in Inches.	Equivalent Length in Centimetres.
1	2·54
4	10·16
10	25·4

Fig. 11 shows that **the graph is a straight line.**

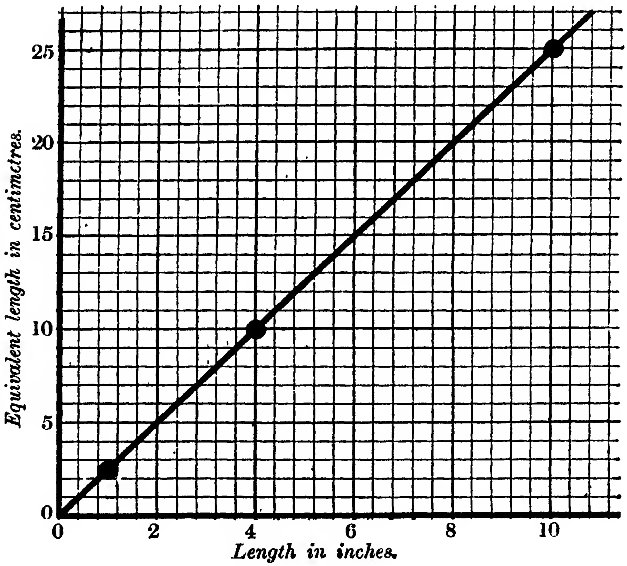

Fig. 11.

Graph for Converting Cubic Feet into Gallons.

Ex. 24. *Given that 1 cubic foot = 6·24 gallons, draw a graph for converting volumes between 0 and 10 cubic feet into gallons.*

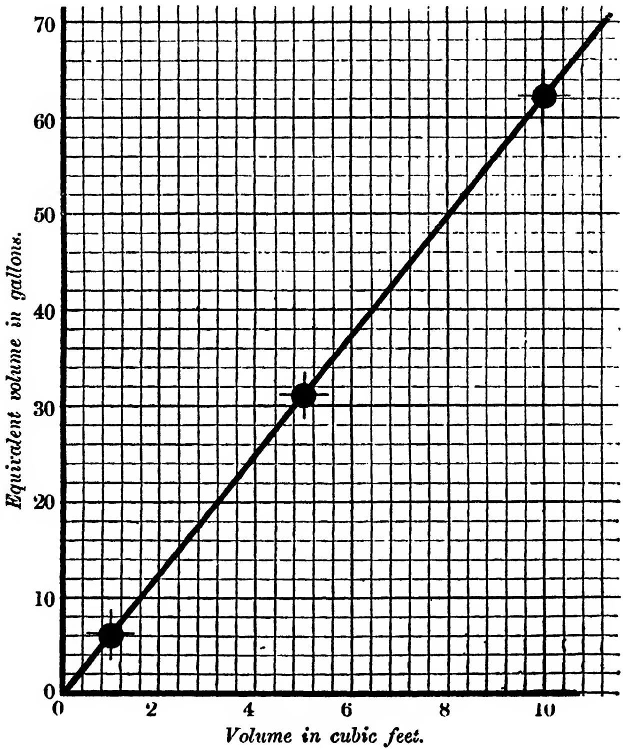

Fig. 12.

From our experience of Example 23 it might be safely inferred that this graph is a straight line. However, we will make quite sure by testing it with a third point.

TABLE OF VALUES.

Volume in Cubic Feet.	Equivalent Volume in Gallons.
1	6·24
5	31·2
10	62·4

Fig. 12 shows the graph.

The student should notice that in the graphs just plotted the scales are made to suit the numbers with which we had to deal. The larger the scale the more accurate the interpolation becomes. Very useful are the exercise books now sold in which alternate pages are of squared paper. These pages are about 9″ by 7″, and the student should aim at making each graph as nearly as possible fill one page.

Exercises 4a.

1. Given that 1 cubic inch equals 16·4 cubic cm., plot a graph for converting volumes between 0 and 10 cubic inches into cubic centimetres. How many cubic centimetres are there in 7·4 cubic inches, and what is the equivalent of 100 c.cm. in cubic in.?

2. A spiral spring is tested by placing weights on it and finding the amount it compresses. The results of the test are given below:—

Load in lb.	2	4	6	8	10	12
Amount of compression in inches	0·14	0·28	0·42	0·56	0·70	0·84

Determine (1) the amount of compression for loads of $2\frac{1}{2}$, $6\frac{1}{4}$, and $11\frac{1}{2}$ lb. respectively. (2) The load required to produce a compression of $\frac{1}{2}''$.

3. The following table gives the volume of a given weight of air at a number of temperatures. Find what the volume would be at 30° C., and the temperature at which the volume would be 15·5 c.c.

Temperature.	Volume in Cubic Centimetres.
0° C.	12·4
10°	12·85
20°	13·3
40°	14·2
60°	15·1
80°	16·0
100°	16·95

4. In bolting two pieces of wood together the pull in lb. was found for every 45° turned through by the spanner. The readings are given in the table:—

Pull in lb.	6·5	13·0	19·5	32·5	45	52
Degrees turned through by the spanner ...	45	90	135	225	315	360

Determine the probable pull in lb. when the spanner has turned through 180° and 270° respectively.

5. A nail was driven into an oak board for various depths and the pull in lb. required to draw it out was found:—

Force required in lb. ...	118	246	370	182	308
Distance the nail penetrates the wood in inches ...	$\frac{1}{2}$	1	1·5	0·75	1·25

Determine the force in lb. required to draw the nail out for distances of $\frac{7}{8}''$ and $1\frac{1}{8}''$ respectively.

6. The following table gives three temperatures on the Centigrade scale of temperature with the corresponding readings on the Fahrenheit scale:—

Centigrade.	Fahrenheit.
0°	32°
50°	122°
100°	212°

Plot a graph and determine the temperature recorded by a Fahrenheit thermometer when the Centigrade reading is 25°, and also the Centigrade record when the Fahrenheit reading is 180°.

7. In a catalogue I find that the price of belting against various widths is given as follows:—

Width in inches	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$
Price per foot in pence ...	3	$3\frac{3}{4}$	$4\frac{1}{2}$	6	$6\frac{3}{4}$	$7\frac{1}{2}$	9	$9\frac{3}{4}$	$10\frac{1}{2}$

Determine the probable prices for $1\frac{3}{4}$, $2\frac{3}{4}$, and $3\frac{3}{4}$ inch widths respectively.

8. In Nettlefolds' catalogue we find that $\frac{5}{16}''$ dia. brass cotter pins are listed as follows:—

Length in inches ...	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	6
Cost in shillings and pence per gross ...	20/6	26/6	32/6	38/6	44/6	50/6	62/6	74/6

Find by plotting the graph what would be the cost for cotter pins whose lengths are respectively $2\frac{1}{4}''$, $3\frac{1}{4}''$, $4\frac{1}{2}''$, and $5\frac{1}{2}''$.

9. The following information is supplied respecting square bars of steel one foot long:—

Length of the side of square...	1″	$1\frac{1}{2}''$	2″	$2\frac{1}{2}''$	3″	$3\frac{1}{2}''$	4″
Weight in lb.	3·4	7·65	13·6	21·25	30·6	41·65	54·4

Plot a graph and determine the weight of bars whose sides measure respectively $3\frac{1}{4}''$ and $4\frac{1}{4}''$.

10. A pendulum is made of a bullet attached to a light thread. Its period (*i.e.* the time in seconds of one complete swing to and fro) is determined for different lengths of the thread. Here are the results:—

Length in Centimetres.	Period in Seconds.
20	0·90
40	1·26
60	1·54
80	1·79
100	2·01

Find the period when the length was 50 cm. and the length required for a period of 1 second.

CHAPTER V.

RATIO AND PROPORTION.

Ratio.—We often meet the idea of ratio in everyday life, though probably we do not give it a name. Thus we say that one object is "twice as big as another," whereby we mean that the ratio of their sizes is as 2 is to 1 (generally written 2 : 1). Or again a tree may be "half as high again" as another; in other words, the ratio of the height of the one to that of the other is as $1\frac{1}{2}$ is to 1; this is the same as the ratio of 3 : 2. We might speak of the ratio of 1″ to 1′ as being $\frac{1}{12}$, or as 1″ is to 12″, whilst the ratio of 1′ to 1″ will be 12, or as 12 is to 1.

It will be noted from the above that we are comparing mentally the magnitude of like quantities, and we may therefore define **ratio** as *the relative magnitude of two quantities of the same kind.*

Fig. 13 shows a graph plotted giving the relation between miles and kilometres. Now if we take a point on the graph representing a distance of 1 kilometre and drop a perpendicular on to the kilometre scale, we form a right-angled triangle in which the perpendicular represents the miles (marked m in the figure), and the base represents kilometres, marked k.

We speak of the graph as showing the relation between miles and kilometres, and, taking the point we have selected, 1 kilometre is equivalent to 0·6213 miles; thus there are 1·609 kilometres to the mile. This might be called the **ratio** of numbers representing kilometres and miles.

Select another point of the graph, that marked P for example. Drop the perpendicular as before, and another right-angled triangle is formed. This time we note from the graph that at 5 kilometres the miles are represented by 3·1068, which is again $\frac{5}{3 \cdot 1068}$, or 1·609 kilo. to the mile.

The student should draw the graph on a large scale, and satisfy himself that the ratio is always the same no matter where he takes the point.

In Example 21 of Chapter IV. we plotted a graph showing the relation between the load and the extension of a wire being stretched (or in tension as it is called). Now if we take a point on the graph representing the extension of 0·1″, we find that the corresponding load is 2·5 lb.; thus the load is numerically 25

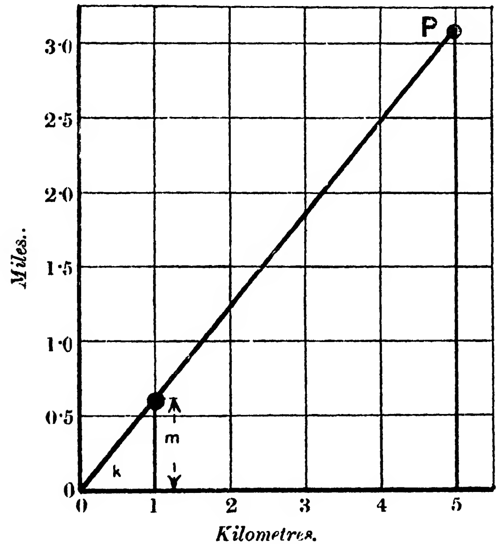

Fig. 13.

times the extension. Here again we have a ratio of numbers representing load and extension. Selecting other points on the graph the student will be able to satisfy himself that the ratio is always the same, and he should turn back to other straight line graphs and apply the same test.

Finally, he should find the ratio of the two quantities (which are often called the variables) whose relationship is shown by a curved line, taking a number of points as before. He will then

see that *when the ratio of two variables is a constant their graph is a straight line.*

It is necessary that we should avoid falling into a common error of speaking of the ratio of the load to the extension. The example discussed above dealt with the ratio of the *numbers* representing the load and the extension. It is quite obvious that we cannot express a relationship between two *unlike* things, while we *can* have a relationship between the *numbers* which represent the magnitude of those things.

Proportion may be defined as *equality of ratios.* Thus if we select any two loads on the graph in Example 21 of Chapter IV., we shall see that the ratio between the two numbers representing these loads will be the same as that between the two numbers representing the corresponding extensions. We may, therefore, say that extension is proportional to load.

In Example 25 the student should draw the figure here described, and follow the reasoning in the text, using measurements made on his own drawing.

Ex. 25. *Illustration of Proportion.*

Draw any triangle (Fig. 14) ABC and measure to 0·01″ the lengths of the three sides AB, AC, and BC. Produce the line BA to D, making AD equal to $2AB$, and produce the line CA to E, making AE equal to $2AC$. Now join DE and measure the length DE. What is the ratio of AC to CB $\left(\text{written } AC:CB \text{ or } \frac{AC}{CB}\right)$? Is it the same as the ratio of AE to ED?

If it is, then we have an equality of ratios (called a proportion), and there are two ways of writing this, *i.e.*

(i) $$AC:CB::AE:ED,$$

which may be read

AC is to CB as AE is to ED,

or (ii) $$\frac{AC}{CB} = \frac{AE}{ED}.$$

This the student has found to be true in one case by direct measurement. Again, what is the ratio of $AB:BC$? Is it the same as the ratio of AD to DE, or, in other words, is this proportion true :—

$$AB:BC::AD:DE?$$

If it is, then the larger triangle, ADE, and the smaller, ABC, are called **similar triangles**. The student should repeat the experiment with several differently shaped triangles.

He should now try to make a pair of "Proportional Compasses." For these he will require two thin pieces of material of exactly equal length, pointed at the ends. These must have a narrow slot down the middle of each piece, and a bolt and nut to slide in the slot to fix them in position. Always keep the legs at each side of the bolt of equal length (Fig. 14). The lengths *CB* and *ED* vary according to the position of the bolt *A*.

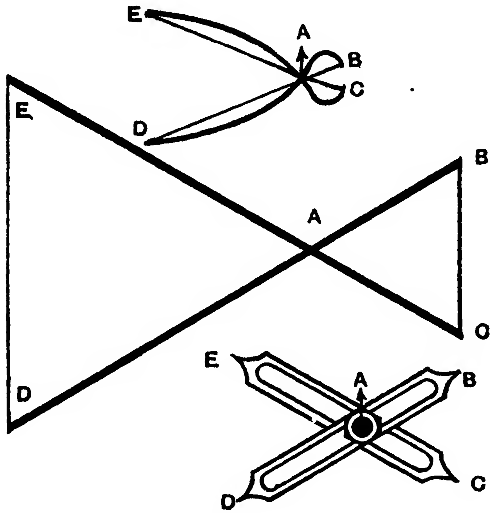

Fig. 14.

For instance, if $EA = AC$ and $AB = AD$, then $BC = ED$. If A is fixed $\frac{1}{3}$ along DB, then $ED = 2\,CB$, etc.; the student must prove this on his model. These proportional compasses are useful in making scale measurements.

Now take a pair of pincers or pliers and find by measurement the value of AB, BC, and EA. Find the value of ED and check it by measurement.

Percentages.—Let us return again to Example 21 of Chapter IV., from which we see that a load of 6 lb. caused a piece of wire to stretch 0·24″. This is nearly $\frac{1}{4}$″ (0·25″), and we should

consider that amount very large indeed if the piece of wire were 1″ long, and, on the other hand, it would be a small extension on a piece of wire 100 ft. long. Whether it is much or little depends not only on its actual amount, but on its *ratio to the whole length.*

The length of wire upon which this experiment was made was 8 ft., so that, with a load of 6 lb., 8 ft. (or 96″) of wire stretched a $\frac{1}{4}''$ (equals 0·25″). In all cases of this kind it is most convenient to consider what would have happened to 100 units. Here 96 inches of wire stretched 0·25″.

Therefore 1″ of wire would have stretched $\frac{0{\cdot}25''}{96''}$, and

$$100'' \text{ would have stretched } \frac{0{\cdot}25 \times 100}{96} \text{ equals } \frac{25}{96}.$$

Converting this to a decimal, we have a little over 0·26. We might, therefore, say that this wire stretched 0·26″ per 100″, or, to adopt the usual term, 0·26 per cent., which is written 0·26 %.

Ex. 26. *A length is measured and found to be* 127·42 *cm., and it is known that it is not more than* 0·15 *per cent. in error. What are the limits within which the true length lies?*

In a length of 100 cm. the error may have been 0·15

Therefore in a length of 1 cm. the error may have been $\frac{0{\cdot}15}{100}$,

and in a length of 127·42 cm. the error may have been

$$\frac{0{\cdot}15 \times 127{\cdot}42}{100} = 0{\cdot}19 \text{ cm.}$$

Therefore the length may be 0·19 cm. longer or shorter than the measured length. That is, the true length lies between 127·23 and 127·61 cm.

Ex. 27. *A test is made upon a large single cylinder gas engine, and gives the indicated horse power as* 117 *and its brake horse power as* 97. *Find the **efficiency** of the engine.*

The *Brake Horse Power* represents the rate at which it will do *work*, and the *Indicated Horse Power* represents the rate at which work is being done in the engine. In other words, the Indicated Horse Power is the rate at which energy is being taken from the coal-gas, etc., in the cylinder, and the Brake Horse Power is the rate at which energy is being taken from the fly-wheel. The latter is always the smaller because of the

energy absorbed in overcoming the friction of the moving parts of the engine.

The Efficiency of an engine is the ratio of the work put into the engine to that which is used. In other words,

$$\text{Efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}}.$$

It is usually expressed as a percentage, in which case the fraction given above must be multiplied by 100. Let us think of it in another way.

Out of 117 units of work we use 97 units.

Therefore out of 1 unit of work we use $\frac{97}{117}$,

and out of 100 units we use $\frac{97}{117} \times 100 = 83$ per cent.

Averages.—The following sums of money represent the daily takings (to the nearest £) of a certain shop for one week :—

Monday, £4.
Tuesday, £6.
Wednesday, £7.
Thursday, £5.
Friday, £9.
Saturday, £13.

This brings the total takings for the week to £44. If we divide this amount by the number of days (viz. 6) we obtain £$7\frac{1}{3}$, which amount is called the average or **mean** daily takings. Hence the average or mean value of a given number of quantities is *the sum of the quantities divided by their number.*

In workshop calculations the most important application of this principle is the determination of **the mean height of a curve.**

Ex. 28. *Find the mean height of a semicircle whose diameter is 4″ long.*

Fig. 15 shows the semicircle. The base (or diameter) should be divided into any number of equal parts, say 10, and *midway between each division* a perpendicular should be erected to meet the circumference of the semicircle. These should be numbered and measured.

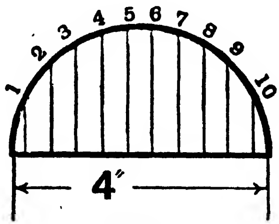

Fig. 15.

If the student draws the figure on squared paper it will greatly facilitate the division, the erection of the perpendiculars, and the measurement of the latter.

The measured lengths of these perpendiculars (or **ordinates,** as they are called) are now set down, and their sum divided by this number. We should notice that in the case of a figure like a semicircle the ordinates numbered 6, 7, 8, 9, and 10 are respectively equal to those numbered 5, 4, 3, 2, 1, and consequently the mean of the first 5 will be the same as the mean of them all.

Number.	Length in inches.
1	0·85
2	1·42
3	1·71
4	1·90
5	1·98
Sum	7·86
Dividing by 5 =	1·57

Therefore the Mean Height = 1·57″.

Exercises 5a.

1. From the data given in Chapter IV. express :—

(*a*) The ratio of 1″ to 1 cm.
(*b*) The ratio of 1 cm. to 1″.
(*c*) The ratio of 1 gallon to 1 cubic ft.
(*d*) The ratio of 1 cubic inch to 1 cubic centimetre.
(*e*) The ratio of 1 sq. ft. to 1 sq. metre.

2. The following readings show the horizontal load required to drag a flat wooden slider along, as different weights were placed on it :—

Horizontal load in lb. = P.	Weight on the slider = W.
1·57	5·7
2·1	7·7
2·66	9·7
3·22	11·7

Determine the ratio between P and W for every case given; express each result as a decimal, and then plot a graph.

3. The following ratio,

$$\frac{1}{\text{Number of threads in one inch}} = \text{Pitch of a screw.}$$

Measure the pitch for the two figures (a and b) given in Fig. 16, and express both as a decimal. If screws have 9, 8, 6, 5, and $4\frac{1}{2}$ threads per inch respectively, express these as a ratio, *i.e.* give their "pitch." Give the answers both as fractions and as decimals.

Fig. 16.

4. The following table gives the distances round circles whose diameters are given :—

Diameter of circle	1″	2″	3″	4″	5″
Distance round the circle called the "circumference"	3·14″	6·28″	9·42″	12·56″	15·7″

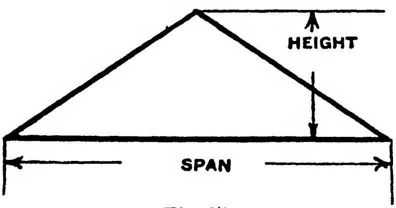

Fig. 17.

Plot a graph and state what is the ratio of $\frac{\text{circumference}}{\text{diameter}}$. Express this both as a fraction (in its simplest form) and as a decimal for each case given in the table.

5. The sketch (Fig. 17) shows that the ratio of $\frac{\text{height}}{\text{span}}$ may be called the "pitch" of a roof. Express the ratio in its simplest form for the dimensions given—

(*a*) when the span is 95 ft. and the height 15 ft.
(*b*) when the span is 100 ft. and the height 20 ft.

6. Draw a square with 2″ sides. Measure the diagonal (*i.e.* the line joining the opposite corners), and express its length as a ratio of the length of a side. Repeat the test on another square. Is it the same?

7. The sketch (Fig. 18) shows the taper shank of a drill. What is the taper per foot and per inch?

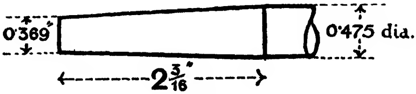

Fig. 18.

8. The sketch (Fig. 19) shows the measurements of a Taper Reamer. What is the taper per foot? Convert the readings into millimetres and find the taper in 30·48 cm.

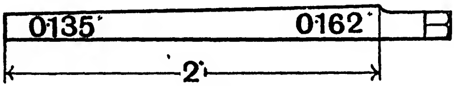

Fig. 19.

9. In erecting a building it is known that the sandy soil on which the foundations are to rest will safely carry $1\frac{1}{2}$ tons per square ft. What area will be required to carry 100,000 lb?

10. Draw any triangle *ABC*. Join the middle points of the sides *AB* and *AC*. What is the ratio of the length of this line to that of the side *BC*? Repeat the problem, using another triangle of entirely different shape. Do you think it is the same for all triangles? Try to draw a triangle in which the ratio would be different.

11. In testing some cement 135 grams were weighed out and sifted through a sieve; 1·52 were caught on the sieve. Express this result as a percentage. On putting the same quantity through another sieve 24·97 grams were caught. Find this percentage.

12. A shopkeeper makes a profit of $33\frac{1}{3}$ per cent. on all articles. Draw a graph showing the relation between the cost price and the sale price of articles selling for less than £2. From the graph find the sale price of an article costing 25s. and the cost price of an article selling for 16s.

13. In testing a piece of wrought iron in tension it is found that the original length of 8″ extends until it is 12·1″ long. What is the percentage extension on the original length?

14. An Insurance Company will insure furniture against damage by fire for an annual payment of 0·1 per cent. of the value of the furniture. What would the payment be for furniture which cost £450. If a fire occurred destroying 70 per cent. of the furniture, how much compensation would the Insurance Company pay?

15. In testing some neat cement the tensile strengths of 6 neat briquettes were respectively 650, 645, 610, 580, 670, and 555 lb. per sq. inch. What is the average tensile strength in this and the following case? Six briquettes made of one part of cement and 3 parts of sand, 250, 246, 255, 251, 262, and 278 lb. per sq. inch.

16. Draw a figure to the dimensions given (Fig. 20), and determine its average height.

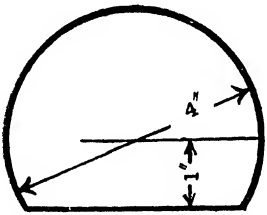

Fig. 20.

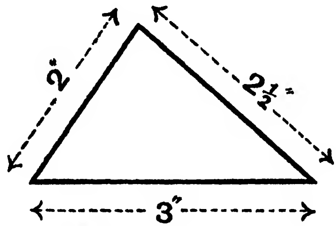

Fig. 21.

17. Draw a triangle to the dimensions given (Fig. 21), and determine the average height. (The method of drawing is described in Example 30, Chapter VI.)

18. Trace the indicator card given (Fig. 22) into your note-book, divide the base into 10 equal parts; measure the middle height of each part and find the average height.

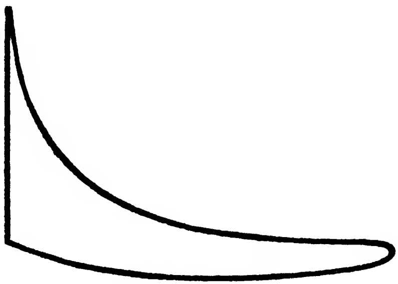

Fig. 22.

CHAPTER VI.

ANGLES AND PLANE FIGURES.

Angles.—Fig. 23 shows a circle, with its centre at *O*. A line drawn from *O* to the boundary (called the circumference) is known as a **radius** (plural radii). Consider the radius *OA*. If it rotate about the point *O* after the manner of the hand of a clock, but in the opposite direction, it will trace out what is called an angle. For example when it has got into the position of *OB*

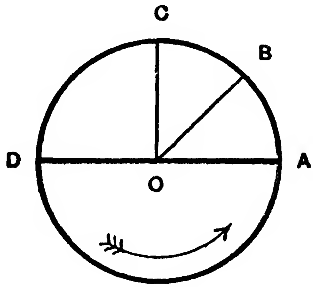

Fig. 23.

it has traced out the angle *AOB*. When the radius has got to *OC* it has travelled through a quarter of the whole circle, and the angle *AOC* is called a **right angle** and the line *OC* is said to be **perpendicular** to the line *OA*.

In the right angle, then, we have a unit of angular measure, but it is too large for practical purposes, so it is divided into 90 equal parts, each of which is called a **degree.** We thus see that there are 360 degrees in a circle, and that an angle of 180 degrees is a straight line, as *AOD* in Fig. 23. (An angle, *e.g.* sixty degrees, is usually written 60°.)

The student should now make for himself a **protractor**, as an angle-measurer is called.

Ex. 29. *Draw a semicircle of $1\frac{1}{2}''$ radius and subdivide into angles not larger than 15 degrees.*

Draw AB (Fig. 24) $3''$ long and mark the middle point O with a $\wedge$. With this as centre and radius $1\frac{1}{2}''$ draw a semi-circle. If the radius is now stepped out by means of the compasses round the circumference, marking the points C and D, it will be found that the semi circle (that is 180°) has been divided into 3 equal parts, each of which is therefore 60°. One of these should now be bisected by placing the point of the compasses on B and making an arc (of any radius more than $\frac{1}{2}BC$), and, with C as centre and the same radius, making another arc to cut the first. If the point of intersection is joined to O the angle of 60° will be bisected. This will fix the angle 30°. This angle should now be bisected in the same way, thus getting an angle of 15°. By stepping out the arc corresponding to this angle round the circumference, the positions marked in the figure are obtained. These should all be joined to the point O.

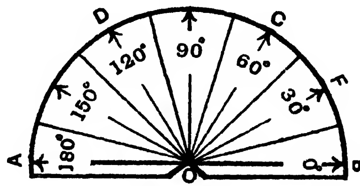

Fig. 24.

To Measure an Angle, place the protractor with the point O on the apex of the angle and with the line OB along one arm of the angle. The position of the other arm can then be read off on the scale. If the arms of the angle are too short to come to the edge of the protractor they should be produced before measurement is commenced.

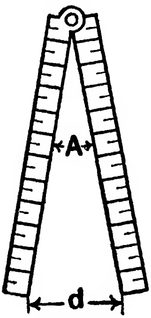

Fig. 25.

To Draw an Angle of a given number of degrees, place the protractor on a line with the point O on one end, and the line OB along the line. A point is now made upon the circumference of the circle at the position of the number of degrees required. This point is then joined to the position of O.

Angles in the Workshop.—For setting out or measuring angles in the workshop it is very convenient to make use of a property of a two-foot folding rule such as carpenters use. (The same results could be obtained with two $12''$ steel scales, common amongst engineers.) If the rule is opened as shown in Fig. 25, the angle (A) at the junction depends upon the distance marked d. The

following table gives the distances corresponding to a number of angles. To obtain intermediate values, the student should plot a graph and obtain the required values by interpolation.

Angles.	Distance.
5°	1·05
10°	2·09
15°	3·13
20°	4·17
30°	6·21
40°	8·21
45°	9·20
50°	10·12
60°	12
70°	13·76
80°	15·43
90°	16·97

Plane Rectilinear Figures.—*A plane figure* is an enclosed flat surface of any shape. If the surface is bounded by straight lines, it is called a *rectilinear* plane figure. Of the rectilinear figures, the most important are **triangles and quadrilaterals,** the former are bounded by three sides and the latter by four.

Triangles are subdivided thus:—an **equilateral** triangle has all its sides equal.

An **isosceles** triangle has two equal sides. A **right-angled triangle** has one angle of 90°.

Quadrilaterals are classified thus:—A **square** has all its sides and angles equal. (The latter are necessarily right angles.) A **rectangle** has all its angles right angles. (Its opposite sides are necessarily equal and parallel.) A **parallelogram** has its opposite sides parallel. A **rhombus** has all its sides equal.

Ex. 30. *Draw a triangle whose sides are respectively* 2·37″, 1·83″, 2·51″.

This triangle is shown in Fig. 26. It should be drawn by setting down one of its sides, *e.g.* BC 1·83″. Then with B as centre and radius 2·51″ an arc is drawn, and with C as centre and 2·37″ as radius another arc is drawn to cut the former. The point of intersection is the point A.

It is convenient to speak of the angles of this triangle as A, B,

and C, and of the sides as a, b, and c, the side a being that which is opposite the angle A and so forth.

The **perimeter** of any figure is the length of the boundary. In a triangle this is the sum of the sides, and Fig. 26 has a perimeter of

$$1{\cdot}83'' + 2{\cdot}37'' + 2{\cdot}51'' = 6{\cdot}71''.$$

If the data for the construction of a triangle contain an angle, this angle should *always* be set out first. The student will find it helpful always to *sketch* the triangle first, and mark the parts which are given.

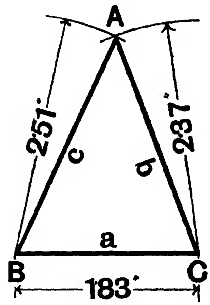

Fig. 26.

Exercises 6a.

Draw the following triangles and measure the parts which are not given. In each case find the length of the perimeter and the sum of the three angles.

1. $a = 11$ cm., $b = 19{\cdot}3$ cm., $c = 15{\cdot}8$ cm.
2. $a = 3{\cdot}46''$, $b = 2{\cdot}44''$, $c = 3{\cdot}79''$.
3. $a = 12{\cdot}35$ cm., $b = 18{\cdot}61$ cm., $C = 55°$.
4. $a = 4{\cdot}65''$, $c = 7{\cdot}81''$, $B = 55°$.
5. $b = 6{\cdot}5''$, $c = 7{\cdot}3''$, $B = 58°$.
6. $a = 12{\cdot}5$ cm., $c = 13{\cdot}8$ cm., $C = 48°$.

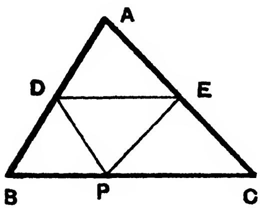

Fig. 27.

The sum of the Three Angles of any Triangle is 180°. This fact is easily proved as follows:—Take a triangular piece of paper as ABC in Fig. 27. Fold it across a line DE, where D and E are the middle points of AB and AC respectively; the point A will just touch the base BC at a point P. Then fold the angles C and B to meet at P also. The three angles of the triangle will thus have met at a point and exactly formed a straight line, *i.e.* 180°.

To draw a quadrilateral we must know at least *one* of the angles, unless one of the diagonals is given in addition to the sides.

Ex. 31. *Draw a quadrilateral ABCD in which $D = 65°$, $AB = 7$ cm., $BC = 12$ cm., $CD = 11$ cm., and $DA = 8$ cm.* (See Fig. 28.)

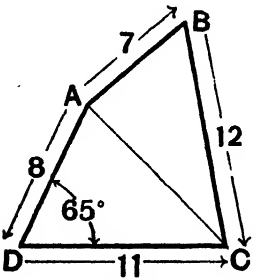

Fig. 28.

First make a rough sketch of the figure and then set out the angle of 65° with a protractor. Upon the arms of this angle the lengths of 11 cm. (DC) and 8 cm. (DA) should be marked off. Then with C as centre and radius 12 cm. draw an arc, and with A as centre and radius of 7 cm. draw another arc cutting the former. This fixes the point B. The student should draw this figure for himself and measure the angles A, B, and C.

It is readily seen that the sum of the four angles of any quadrilateral is equal to four right angles (360°), for if a diagonal is drawn, as AC in Fig. 28, the figure is divided into two triangles, the angles of each of which together make 180°.

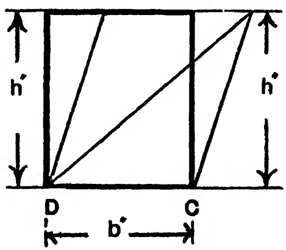

Fig. 29.

Areas.—If the student draws a rectangle 4″ long by 3″ wide on squared paper, he will see that it contains exactly 12 squares with 1″ sides. In other words its area is 4×3 sq. in. In Fig. 29 a rectangle $ABCD$ is shown on a base of "b" units and having a height of "h" units. Its area is therefore $b \times h$, which is written bh.

In the same figure is shown a parallelogram $EFCD$ standing on the same base DC ("b") and having the same *vertical* height ("h"). Now these two figures are partially superimposed, but there is a portion of the rectangle (viz. the triangle DAE) and a portion of the parallelogram (viz. the triangle CBF) which stand alone. If these be cut out and superimposed they will be found to be equal. The student

should make a number of trials, making the parallelograms more and more slanting. He will find that *parallelograms standing on the same base (or equal bases), and of the same vertical height are equal in area,* that area being the product of the base and the *vertical* height.

It is readily seen that any of these parallelograms might have been divided into two equal parts by drawing a diagonal, each part being a triangle. Thus triangles standing on the same base (or equal bases) and of the same vertical height are equal in area. Hence *the area of a triangle is equal to one half the product of the base and the vertical height.*

If the area of an irregular quadrilateral is required, it is usual to divide it up into triangles and take the sum of the areas of these. Indeed this method may be applied to *any* rectilinear figure, however many sides it may have.

Exercises 6b.

Find the areas of the following figures :—

1. An equilateral triangle with sides 7 cm.

2. An isosceles triangle of base 1·72″ and equal sides of 2·61″.

3. A triangle ABC in which $a = 9$ cm., $c = 8{\cdot}2$ cm., and $B = 55°$.

4. A parallelogram having sides whose lengths are 2·34″, and 3·27″ respectively and one of whose angles is 46°.

5. The quadrilateral described in Example 31.

6. A rhombus whose sides are each 9·2 cm., and one of whose angles is 115°.

Ex. 32. *A kitchen floor is to be covered with square tiles 6″ by 6″. If the floor is 14ft. by 13ft., how many tiles will cover the floor?*

It will be seen that the area of the floor $= 14' \times 13' = 182$ sq. ft.

Each tile will cover an area $6'' \times 6'' = 36$ sq. in. or $0{\cdot}5' \times 0{\cdot}5' = 0{\cdot}25$ sq. ft.

The number of tiles required $= \dfrac{\text{area of floor}}{\text{area of one tile}} = \dfrac{182 \text{ sq. ft.}}{0{\cdot}25} = 728$ tiles.

Ex. 33. *A room measures 20′ by 15′. How many feet of floor boards $7\frac{1}{2}''$ wide and having square joints will be required?*

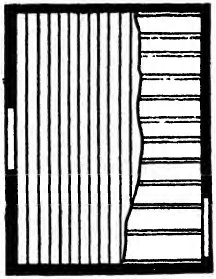

Fig. 30.

Suppose the boards have to be fixed as in Fig. 30. Since the floor is 15 ft. wide = 180″,

The number of boards

$$= \frac{180''}{7\frac{1}{2}''} = \frac{180}{7\cdot5''} = 24 \text{ boards.}$$

Therefore 24 boards laid side by side each 20′ long will be required, or 480′ run of boarding.

The student is advised in problems of this type to make a sketch.

Exercises 6c.

1. A wall 9′ 3″ by 12′ 3″ has to be covered with wall paper which is sold in rolls 35 ft. long, $1\frac{3}{4}$ ft. wide. How many feet will be required (allowing $\frac{1}{7}$ above the calculated quantity for waste)? Also state how many rolls will be required.

2. How many bricks 9″ by $4\frac{1}{2}''$ will be required to pave a path 3 ft. wide and 33 ft. long?

3. A door is to be painted and measures 2′ 4″ by 6′ 6″. Determine the area in sq. feet, adding 25 per cent. to the area found to allow for the framing and moulding.

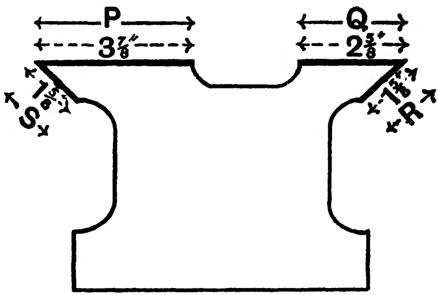

Fig. 31.

4. Fig. 31 shows a section of a lathe bed, the parts marked *P. Q. R. S* have to be machine planed. If the length of the bed

is 6 ft., determine the total area (in sq. in.) which requires planing.

5. 220 compartments are required for storing tools in a space 7′ 4″ long by 5′ high. What space for each compartment will this allow, neglecting the thickness of metal separating the one from the other?

6. The dimensions of a field were taken with a surveyor's chain and are shown in Fig. 32. Determine the area in acres and square metres, using the information given below.

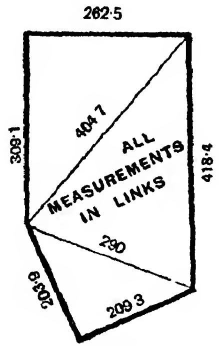

Fig. 32.

1 Gunter's Chain = 66 feet = 100 links.
20 metres = 66 feet approximately (65·6 feet).
1 acre = 10 sq. chains. (The student should find the area in sq. chains and move the decimal point to get the area in acres.)

CHAPTER VII.

CIRCUMFERENCE AND AREA OF A CIRCLE.

Ex. 34. *Find the ratio of the diameter of a penny to its circumference.*

(N.B. The student should do this example, using his inch scale and making measurements correct to 0·01″. The problem will be worked out here, using the centimetre as the unit of length.)

First measure the diameter of the coin by placing the scale across it and by trial finding the widest part.

Diameter = 3·09 cm.

To measure the circumference rule a straight line on a flat piece of paper. Make a scratch on the edge of the penny and place it on the line so that the scratch rests on the end of the line. Now roll the coin along the line, being careful to avoid slipping, until the scratch comes back to the line. Mark this point on the line and measure the length of line traversed. This gives the length of the circumference of the circle.

Circumference = 9·7 cm. The required ratio is given by

$$\frac{\text{Circumference}}{\text{Diameter}} = \frac{9 \cdot 7}{3 \cdot 09} = 3 \cdot 14.$$

The student should repeat this example, using a number of different coins and, if possible, larger discs (such as circular tin lids). Is the ratio the same for all circles? Does it remain unaltered if the measurements are made in other units?

Ex. 35. *Find the ratio of the diameter of a lead pencil to its circumference.*

The diameter may be measured as before. Diameter = 0·29″.

The circumference, however, is too small to obtain its length with sufficient accuracy by the method just adopted in the case of the penny. Take a piece of cotton and wind it neatly round the pencil (like the binding on the handle of a cricket bat). Cut the cotton so as to have an exact

number of turns (say 10) on the pencil, and then unwind the cotton and measure it.

$$\text{Length of 10 turns} = 9{\cdot}12''$$

$$\text{Therefore circumference} = \frac{9{\cdot}12}{10} = 0{\cdot}912''$$

$$\text{Required ratio} = \frac{\text{Circumference}}{\text{Diameter}} = \frac{0{\cdot}912''}{0{\cdot}29} = 3{\cdot}14.$$

The student should repeat this example, using metal rods, gas piping or broom handles.

The most accurate measurements show us that this ratio is constant for all circles. In ancient Greek mathematics it was represented by the symbol π, which is the Greek letter for P and is pronounced "pie," and the same symbol is still used universally. Its value has been determined to a great number of significant figures. The first five are sufficient for all ordinary purposes and are 3·1416.

We therefore see that **the circumference of a circle is 3·1416 times the diameter.** So that if we represent the diameter of a circle by the letter d, we may say the circumference $= \pi d$; or if r represent the radius of a circle, the circumference $= 2\pi r$ (since $d = 2r$).

To find the Area of a Circle.

Ex. 36. *Find the area of a circle whose radius equals r.*

In the upper part of Fig. 33 we see the circle. If its centre be joined to two points *very close together*, on the circumference we shall have a figure which will be more nearly a triangle the nearer the points are taken together. If the circle were divided up into a very large number of these triangles, which were then arranged as shown in the lower part of Fig. 33, we might find the area thus :—Consider one triangle, its area $= \frac{1}{2}$ (base × height) but its height $= r$, therefore its area $= \frac{1}{2}r \times$ (base) and the area of all the triangles $= \frac{1}{2}r \times$ (sum of all the bases) $= \frac{1}{2}r \times$ (circumference of the circle).

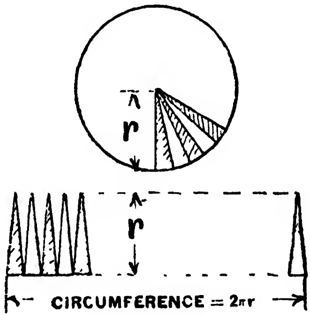

Fig. 33.

$$= \tfrac{1}{2}r\,(2\pi r)$$
$$= (\tfrac{1}{2} \text{ of } 2) \times \pi \times r \times r$$
$$= \pi r^2.$$

This is a very important formula and should be remembered. It is sometimes convenient to have an expression for the area of a circle in terms of its diameter.

$$\pi r^2 \text{ becomes } \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}d^2 = \frac{3{\cdot}1416}{4} \,.\, d^2 = 0{\cdot}7854\, d^2.$$

Ex. 37. *Find the circumference and area of a circle whose radius is* 5 *cm.* (*Three-figure accuracy.*)

$$\begin{aligned}\text{Circumference} &= 2\pi r\\ &= 2 \times 3{\cdot}1416 \times 5\\ &= 31{\cdot}416 \text{ cm.}\end{aligned}$$

$$\begin{aligned}\text{And area} &= \pi r^2\\ &= 3{\cdot}1416 \times 5 \times 5\\ &= 78{\cdot}54 \text{ sq. cm.}\end{aligned}$$

To make a Circumference Gauge.—The student should make a circumference gauge of cardboard or wood. Fig. 34 shows this idea. It consists of a head piece *A* which has a connection *B* sliding into a body *C*. Marked upon the surface *C* against the lower edge of *B* is a scale of inches and tenths, and against the upper edge of *B* a scale which is 3·1416 time the former. A mark is made on *B* such that it is opposite the *O* of both scales when the jaws are closed. If a rod is placed within the jaws, the lower scale gives its diameter and the upper one its circumference.

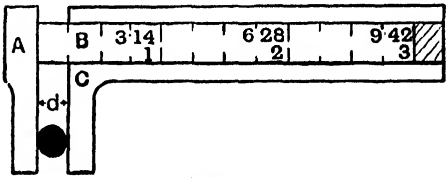

Fig. 34.

Exercises 7a.

1. Find the circumference and area of circles whose radii are respectively 0·2, 0·7, 1·3, 2·41, and 3·65 cm.

2. Find the circumference and area of the circles whose diameters are respectively 0·06, 0·81, 2·46, 4·78, and 5·27 in.

3. Find the diameter and area of the circles whose circumferences are 3·14, 17·25, 23·26, 75·81, and 115·7 cm.

The Semi-Circle.—Draw a semi-circle on a diameter of 3″. Select a number of points on the circumference and join to the extremities of the diameter. Measure the angles formed.

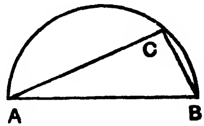

Fig. 35.

Fig. 35 shows *one* of the angles so formed. Measurement shows that this is a right angle, that is 90°. Is it the same for all points? Is the magnitude of the angle affected by the length of the diameter? May we say that **the angle in a semi-circle is a right angle?**

Right-Angled Triangles.—Draw a triangle whose sides are respectively 3, 4, and 5 in. long. Measure the angle opposite the longest side. Then multiply 3, 4, and 5 by any factor, say 0·7; they then become 2·1, 2·8, and 3·5 in. Draw this triangle and again measure the largest angle. Try another factor.

Now $3^2 + 4^2 = 9 + 16 = 25$, and this $= 5^2$.
Also $(2{\cdot}1)^2 + (2{\cdot}8)^2 = 4{\cdot}4 + 7{\cdot}8 = 12{\cdot}2$ which is practically equal to $(3{\cdot}5)^2$

Does this relation hold in other cases where other factors are used? Is the largest angle always a right angle? Draw a number of right-angled triangles and measure their size. Does the above relationship hold in cases where the sides are not in the ratio of 3, 4, and 5?

In drawing a right-angled triangle the largest side (called the **Hypotenuse**) should usually be drawn first and a semi-circle described on it. The right angle then falls on the circumference of this semi-circle. The above examples should have established two very important facts which should be remembered:—

1. When the sides of a triangle are in the ratio of 3, 4, and 5, the triangle is always a right-angled triangle.

2. The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

Ex. 38. *Find the length of a ladder required to reach a window* 50 *feet high when the foot of the ladder is* 10 *feet from the house front.*

Fig. 36 shows that the ladder forms the hypotenuse of a right-angled triangle, and if we call its length l then :—
$l^2 = 50^2 + 10^2 = 2500 + 100 = 2600$. Therefore $l = \sqrt[2]{2600} = 51$ ft. (approximately).

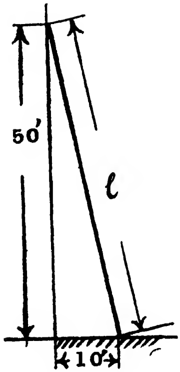

Fig. 36.

Ex. 39. *Find the length of the edge of the largest square shank which can be cut on the end of a 1″ diameter round bar.*

This really resolves itself into finding the largest square which can be drawn in a circle of 1″ diameter. Fig. 37 shows the square, the diagonal of which is of course equal to the diameter of the circle. If we call the length of the side of the square l, then $l^2 + l^2 = 1^2$, *i.e.* $2l^2 = 1$ and $l^2 = \frac{1}{2} = 0{\cdot}5$, $l = \sqrt[2]{0{\cdot}5}$ $= 0{\cdot}71''$ (approximately).

Ex. 40. *Find graphically the square root of* 2, 3, 4, 5, *etc.*

Draw a right-angled triangle ABC in which AB and AC are each 10 cm., see Fig. 38. Calling 10 cm. 1 unit the length of BC gives $\sqrt{2}$. It is found to be 14·2 cm., therefore the $\sqrt{2} = \frac{14{\cdot}2}{10} = 1{\cdot}42$. Mark off $AD = BC$ and join BD. Then $BD = \sqrt{3}$. Mark off $AE = BD$ and join BE, which gives us $\sqrt{4}$ and so forth.

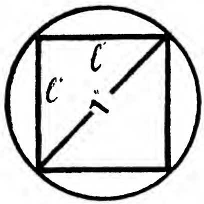

Fig. 37.

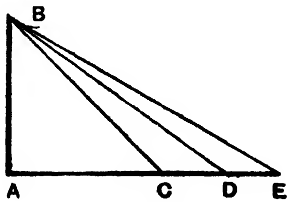

Fig. 38.

Exercises 7b.

1. Wire is sold by gauge (known in England as Imperial Standard Wire Gauge, written S.W.G.). No. 17 has an equivalent diameter of 0·056″. No 38 has an equivalent diameter of 0·006″. Determine the area of the section of these wires in square inches and in square millimetres.

2. A four-cylinder petrol engine has cylinders whose diameters measure 65 millimetres (called the bore). Determine in sq. in. the combined cross sectional area of the four cylinders.

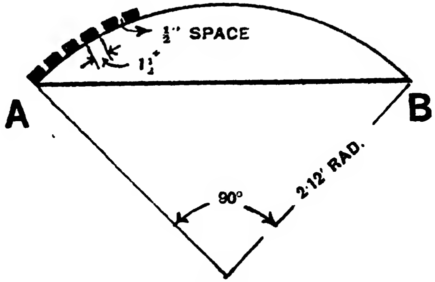

Fig. 39.

3. The diameter of the driving wheels of a locomotive is 5ft. 6 in. If that of the axle at the centre of the wheel is $6\frac{3}{4}''$ determine the distance the wheel travels in one revolution, the circumference of the axle, and the number of turns the wheel makes in a mile.

4. A diagram of a segmental arch is shown in Fig 39. Determine the distance from A to B measured round the curve. (N.B. —Since the angle shown in the diagram is 90° the length of the

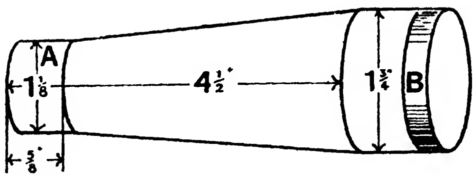

Fig. 40.

arc is $\frac{90}{360} = \frac{1}{4}$ of the circumference of the circle.) Also find how many strips $1\frac{1}{2}''$ wide and spaced $\frac{1}{2}''$ apart will be required to form the lagging. If each of the strips is 13″ long, how many feet of wood $1\frac{1}{2}''$ wide will be required ?

5. A water tap has a diameter of $\frac{3}{4}''$ at the nozzle. Determine the sectional area in sq. ft. and sq. cm.

6. Determine the sectional area of the pipe across A and B (Fig. 40) for the dimension given and give a sketch.

7. Fig. 41 is a sketch of a roof truss. Determine the length of the side AB. If the wind causes a normal pressure of 14·53 lb.

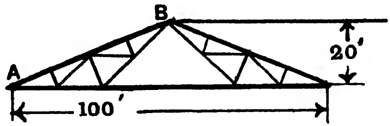

Fig. 41.

on the sq. ft. on AB and acts on an area = length AB × 15 ft., what is the total wind pressure to the nearest lb.?

8. A line sketch is shown in Fig. 42 of a crane. Determine the length of the Jib B and of the Rod A.

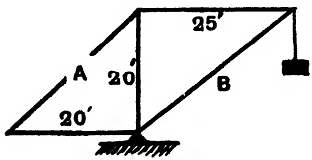

Fig. 42.

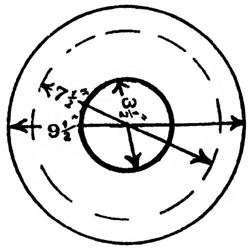

Fig. 43.

9. Fig. 43 shows a pipe flange, and six $\frac{7}{8}''$ holes are to be drilled on the dotted circle at equal distances apart. Calculate the distance (1) measured round the circumference of the circle and (2) measured in straight lines from one to another. Make a sketch showing the holes drilled.

CHAPTER VIII.

USE OF FORMULAE.

We have seen that the area of a circle is obtained by squaring the radius and multiplying by the value of π (viz. 3·1416). If we denote the area by A and the radius by r we may write $A = \pi r^2$. This expression is an **equation** because it tells us that one thing (A) *is equal to* another (πr^2). It might also be called a **formula** (plural formulae), by the aid of which we can calculate A if we know r—and *vice versa*.

There are many formulae in use, some of which have been obtained mathematically, as we obtained $A = \pi r^2$ in Chapter VII. Others are obtained from experimental results. For example in Chapter V., we saw how a straight line graph showed the ratio of numbers representing a force to an extension, and it was seen that on all points on the graph the force (in lb.) was 25 times the extension (in inches). Using the same symbols we might write $F = 25d$.

Here we have a formula, a simple one certainly, that has been obtained from some experimental results. It might now be used to calculate F for any value of d.

We thus see how convenient it is to express experimental or mathematical results by means of formulae. We shall here examine one or two types of formulae and make some calculations by their aid.

Ex. 41. *If D denote the external diameter of a piece of gas piping and d denote the internal diameter, the sectional area of metal is given by $A = 0·7854\ (D^2 - d^2)$.*

Calculate A when $D = \frac{3}{4}''$ and $d = \frac{9}{16}''$.

$D = \frac{3}{4}'' = 0·75''$. Therefore $D^2 = 0·563$ sq. in.

$d = \frac{9}{16} = 0·5625''$. Therefore $d^2 = 0·317$ sq. in.

Therefore $(D^2 - d^2)$ $= 0{\cdot}246$
Multiplying by $\cdot7854$ $0{\cdot}7854$

$$\begin{array}{r|} 0{\cdot}1722 \\ 192 \\ 10 \\ \hline \end{array}$$

$A =$ $0{\cdot}192$ Result $= 0{\cdot}19$ sq. in.

Note.—It would *be wrong* to subtract d from D and then square the remainder. If this operation had been intended it would have been written as $(D - d)^2$.

Ex. 42. *The Indicated Horse Power (I.H.P.) of a gas engine is given by the formula* $I.H.P. = \frac{P.L.A.N.}{33000}$.

Where $P =$ *mean effective pressure on the piston in lb. per sq. in.*
$L =$ *length of stroke in feet.*
$a =$ *area of the piston in sq. in.*
$N =$ *number of explosions occurring in the cylinder per minute.*

Find the I.H.P. of a gas engine of $5{\cdot}5''$ *bore and* $11''$ *stroke, given that* $P = 66{\cdot}8$ *lb. per sq. in. and* $N = 85$.

The diameter of the piston $= 5{\cdot}5''$.

Therefore $A = 0{\cdot}7854\,(5{\cdot}5)^2 = 23{\cdot}8$ sq. in.

and $L = 11'' = \frac{11}{12}$ ft.

Now $$\text{I.H.P.} = \frac{P.L.A.N.}{33000}$$

$$\frac{66{\cdot}8 \times 11 \times 23{\cdot}8 \times 85}{12 \times 33000}.$$

Contracting the multiplication and division so as to obtain three significant figures only we get

$$\text{I.H.P.} = 3{\cdot}75.$$

Dimensions of Machine Parts.—In books on machine design it is usual to give drawings of machine parts or even of a complete engine, on which no actual measurements are placed but only a series of numbers. The dimensions for any particular case are obtained by multiplying these numbers by a unit as it is called.

In Fig. 44 we have particulars of a "Box Coupling" connecting two lengths of shafting. The unit placed upon the drawing

is $(d + \frac{1}{2})$. Here d indicates the diameter of the shaft. Hence, to obtain the dimensions when the shaft has a diameter of $1\frac{1}{2}''$ we must multiply the numbers given by $(1\frac{1}{2} + \frac{1}{2})$ or 2. That is, the length of the coupling must be $3{\cdot}25 \times 2 = 6\frac{1}{2}''$, and the thickness $0{\cdot}48 \times 2 = 0{\cdot}96''$ (say $1''$).

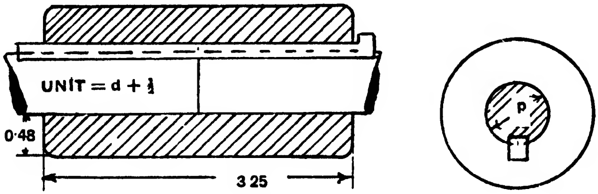

Fig. 44.

If the shaft had a diameter of $2''$ then the factor to be applied would be $(2 + \frac{1}{2})$ or $2{\cdot}5''$, and the length of the coupling would have to be $3{\cdot}25 \times 2{\cdot}5 = 8{\cdot}125''$, and the thickness $0{\cdot}48 \times 2{\cdot}5 = 1{\cdot}20''$.

Exercises 8a.

1. Find the sectional area of a pipe whose external diameter is $1''$ and internal diameter $= \frac{13}{16}''$ (see Example 41 for the formula).

2. Find the length and thickness of the box coupling shown in Fig. 44 suitable for a shaft $1''$ diameter. Make a sketch of the end view of this coupling, and put the dimensions on it. What would the sectional area be?

3. The surface area (S) of a sphere of radius r is given by :—$S = 4\pi r^2$ whilst its volume (V) is given by $V = \frac{4}{3}\pi r^3$. (N.B. r^3 means $r \times r \times r$.) Find the surface area and volume of a steel ball taken from a "ball bearing," the diameter of the ball being $\frac{3}{8}''$.

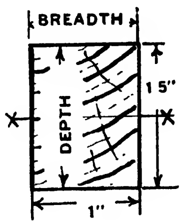

Fig. 45.

4. The Moment of Inertia of a rectangular section about the axis xx (Fig. 45) is given by

$$\frac{\text{Breadth} \times \text{Depth}^3}{12}.$$

Determine the Moment of Inertia of the section given in Fig. 45.

5. The diameter of a rivet is found for roof and other riveted work from the following formula:—

Diameter of rivet $= 1{\cdot}1 \sqrt{\text{thickness of plate}}$.

Find the diameter of the rivet for $\frac{5}{8}''$, $\frac{3}{4}''$, $\frac{7}{8}''$ and $1''$ plates. Give results to the nearest sixteenth of an inch.

6. The Horse Power of a petrol engine is given by the following formulae:—

$$\text{H.P.} = \frac{D^2 \times N}{2{\cdot}5}, \quad D = \text{diameter of the cylinder in in.} \quad N = \text{number of cylinders.}$$

$$\text{H.P.} = \frac{D^2 \times N}{1613}, \quad D = \text{diameter of the cylinder in millimetres.}$$

Determine the Horse Power by both formulae for a four-cylinder petrol engine, the diameter of the cylinders (commonly called the "bore") being 75 millimetres.

7. If a body is thrown vertically upwards with a velocity of v ft. per second, the height S to which it will rise is given by $S = \frac{V^2}{2g}$. Where the value of g depends upon the earth's attraction and is $32{\cdot}2$ at sea level in this country, how high will a body rise if it is thrown vertically upwards with a velocity of 60 ft. per second?

8. The Indicated Horse Power of a double-acting steam engine is given by $\text{I.H.P.} = \frac{2\,.\,P.L.A.N.}{33000}$. Where P, L, and A have the same meaning as in Example 42, and N = number of revolutions per minute, find the I.H.P. of a steam engine of $4''$ bore and $5''$ stroke when $P = 58$ lb. per sq. in. and $N = 280$. (It may be of interest to note that P is the *mean pressure* throughout the stroke and is always much less than the gauge pressure of the boiler.)

9. If a, b, c are the lengths of the sides of a triangle and S represents half the perimeter, the area (A) of the triangle is given by:—

$$A = \sqrt{S\,(S-a)\,(S-b)\,(S-c)}.$$

(N.B.—The four expressions under the root sign having been

substituted for their values should be *multiplied* together and then the square root extracted.) Find the area of a triangle whose sides are 5, 6, and 7 cms. Draw the triangle and find its area by the method described in Chapter VI.

10. If r represents the electrical resistance of a wire at a temperature of 0° C. and R be its resistance at t° C., then $R = r(1 + at)$ where a is a constant depending on the nature of the metal. For pure metals it is about 0·0038. If a piece of copper wire have a resistance of 5 ohms (the ohm is an electrical unit of resistance) at 0°, find the resistance of the wire at 100° C.

11. The area of an ellipse is given by $A = \pi ab$ where a and b are the semi-major and semi-minor axes (*i.e.* half the longest and shortest axes respectively). Find the area of an ellipse whose major axis is 3·15 cm. and whose minor axis is 1·74 cm.

12. The following formula gives the amount of the super elevation (AC, Fig. 46) of a railway track:—

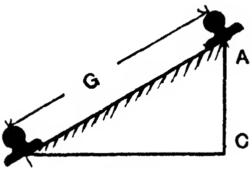

Fig. 46.

$$AC = G \cdot \frac{v^2}{32r}$$ where $G = 4'\ 8\frac{1}{2}''$ (the English railway gauge),

v = velocity in feet per second,

r = radius of the curve in feet.

Determine the super elevations for a curve of 250 ft. radius for velocities of 8·8 ft. per sec., 13·2 ft. per sec., and 17·6 ft. per sec.

(The student should observe the banking up of cycle and motor tracks.)

CHAPTER IX.

ALGEBRAIC PROCESSES.

Algebra, so far as it enters into work-shop calculations, is that branch of mathematics dealing with symbols which represent measurable quantities.

Everyone in this country is accustomed to the idea of the letter *d* representing a penny, and consequently 3*d.* means a value three times that of a penny. In an expression such as 3*d.* the quantity 3 is known as the coefficient. Quite young children quickly learn to do "money sums" without troubling themselves about the real value of a penny. Its value may vary from time to time; in fact, we are told that its purchasing power is less to-day than it was ten years ago. The variable value of the penny however never influences a child's "money sums," and in just the same way algebra deals with symbols without giving a thought to any particnlar value which each symbol may possess.

Algebraic Addition.—Just as *d* represents a penny, so we let *s* and *l* represent one shilling and one pound respectively. Now a sum of money could be expressed thus:—$3l. + 5s. + 7d.$ This is generally written £3 . 5 . 7, the positive signs and the indication of shillings and pence being so obvious that they are often omitted. Now suppose that, to the amount mentioned above, it is necessary to add:—$10l. + 18s. + 3d.$ and $14s. + 8d.$, the problem would be set out thus:—

$$\begin{array}{rrr} 3l. + & 5s. + & 7d. \\ 10l. + & 18s. + & 3d. \\ & 14s. + & 8d. \\ \hline 13l. + & 37s. + & 18d. \end{array}$$

We instinctively avoid placing the 14*s.* in the "pounds column,"

because it is obvious that shillings and pounds are *unlike quantities*. We cannot express the sum of $10l.$ and $14s.$ in a single term, because it would be wrong to call the sum $24l.$ and equally wrong to call it $24s.$ The only way of expressing such a sum is $10l. + 14s.$ Thus we see the importance of placing only like quantities in the same column.

In the case of an addition of money it is not usual to leave $18d.$ in the pence column, but to place $6d.$ there, and carry one shilling forward to the shillings column. It is possible to do this because it is known that $1s. = 12d.$, but in the majority of algebraical expressions no such relationship is known, and consequently the result would have to remain as it is in the foregoing example.

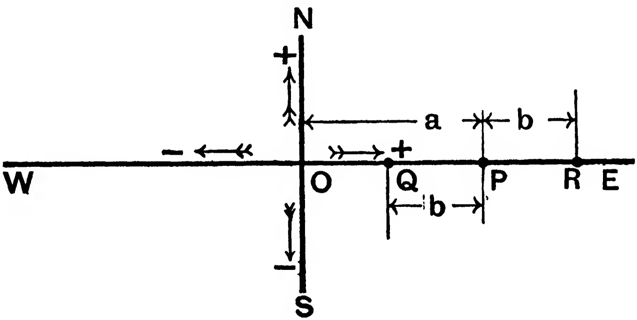

Fig. 47.

We must always remember that the letters used in Algebra represent *numbers* (often of unknown magnitude). In the example just considered we must avoid thinking of l, s, and d representing coins of money, but think of them as standing for *numbers*. Thus if we regard the penny as the unit, then $d = 1$, $s = 12$, and $l = 240$.

It has been shown in Chapter V. how a length may be made to represent any kind of quantity. We have grown accustomed to the idea of *minus quantity*, let us now consider a *minus distance*. In Fig. 47 the four cardinal points of the compass are marked. Consider a man starting from the point 0 (called the **origin**). It is convenient to regard his motion as positive if he move towards the north or east, and negative if he move

towards the south or west. Suppose he walks towards the east (*i.e.* in the positive direction) for a distance a yards, this will bring him to the point P. If he now continues his journey in the same direction for a distance b yards he will arrive at the point R. His distance from 0 is now $(a + b)$ yards. If, however, he had paused at the point P and *turned round* before proceeding with the second portion of his journey, the distance b yards would have been accomplished in the *negative direction*, and the end of his journey would have been at Q, a distance of $(a - b)$ yards from the origin.

Let us now think of a man starting from 0 and facing east with instructions to walk $a - (-b)$ yards. He will walk as far as P (*i.e.* a yards). The minus sign causes him to turn round and face west. Inside the bracket, however, is another minus sign which tells him to *turn round* again. This brings his face to the east, and he proceeds on his journey for b yards which brings him to R, a distance of $(a + b)$ yards from his starting point. In other words :—

$$a - (-b) = a + b.$$

This is a very important fact, and the commonest way of expressing it is :—"Two minuses make a plus."

In algebraical work it is very convenient to group isolated quantities together by means of brackets, thus :—

$$(2a + 3b - 6c) + (4a - 3c - 4b) - (7b + 6a - 8c).$$

Here we have instructions to add the expression contained in the first bracket to that contained in the second bracket, and from the sum so obtained to subtract the expression contained in the last bracket. The problem should be set out thus :—

1st expression..........................	$2a + 3b + 6c$
2nd expression	$4a - 4b - 3c$
Sum	$6a - b + 3c$
3rd expression	$6a + 7b - 8c$
Difference	

(Note that the a's and b's and c's have been placed in their respective columns.)

In finding the sum we have in the left hand column $(2a + 4a)$. This obviously becomes $6a$, just as 2 pence + 4 pence equals

6 pence. In the next column there is $(3b - 4b)$. Suppose b to represent some unit of distance, then $3b$ signifies 3 units towards the east and $-4b$ means 4 units westward (*i.e.* back again), which would bring the traveller 1 units west of the origin (*i.e.* $-1b$). The figure 1 is usually omitted in writing the expression, it being understood that $-b$ indicates $-1b$. In exactly the same way the last column $(6c - 3c)$ results in $+3c$.

Algebraic Subtraction.—We will now consider the subtraction part of the problem, commencing with the left hand column. The instruction to subtract will be given by the minus sign.

1st column .... $6a - (6a)$

This is $6a$ units to the east and $6a$ units back again. *Result*, 0.

2nd column .... $-b - (7b)$

In this case both quantities are negative, *i e.* both distances are performed in the same direction. Thus, $-b - 7b = -8b$.

3rd column .... $3c - (-8c)$.

This is $3c$ units to the east. The first minus sign brings the man facing west, and the second minus sign turns him round again so as to face east; then the journey is continued for a distance of $8c$, making $11c$ units in all to the east. *Result*, $11c$.

The final result of the problem is therefore $-8b + 11c$.

We are now in a position to formulate the rules of algebraic addition and subtraction.

Rules for Algebraic Addition.—(1) *Arrange the expressions so that only like quantities occur in each column.*

(2) *Add all the positive coefficients together, then all the negative coefficients. Take the difference between these two sums and give the sign of the greater.*

Rules for Algebraic Subtraction.—(1) *Arrange the two expressions with their like quantities under each other, taking care to place the expression* **from which** *the subtraction has to be made on the top.*

(2) *Mentally change the signs of the terms in the bottom line and proceed as in addition.*

Exercises 9a.

1. Arrange like quantities in columns and add together:—

(i) $5x + 3y + 8z + 10x + 2y + 3z + 4x + 16y$.

(ii) $5a + 2b + 3 + 12a + 10b + 10 + 2a + b + 1$.

2. Explain graphically the meaning of the following:—

(i) $A + B + 2C$.

(ii) $3A + 2B + C$.

(iii) $A + 5B + 3C$. Take as values, $A = \frac{1}{2}''$, $B = \frac{1}{4}''$, $C = \frac{3}{4}''$.

3. Arrange like quantities in columns and add together:—

(i) $7 + x + 9y + 10x + 3 + 2y + 4y + x + 1$.

(ii) $6 + S + 2W + 5S + 7W + 2 + 4 + 4S + 4W$.

4. Explain graphically the meaning of:—

(i) $a/10 + b/6 + c/4$.

(ii) $a/1 + b/2 + c/2$.

(iii) $a/4a + b/8 + c/16$. Take as values:—$a = 1''$, $b = 2''$, $c = 4''$.

5. Add together

$$
\begin{aligned}
&a/5 + 2b + c.\\
&a/10 + b + 2c.\\
&a + 4b + 3c.
\end{aligned}
$$

If $a = 1$, $c = \frac{1}{2}$, $b = \frac{3}{4}$, find their numerical value.

6. Find the numerical values in shillings of the following expressions, take as values, $x = 1/\text{-}.$, $y = 6d.$, $c = 3d.$, $d = 1d.$

(i)

$$
\begin{aligned}
5x &+ 2y + 3c + 4d.\\
2x &+ y + c + 2d.\\
x &+ 3y + 2c + 4d.
\end{aligned}
$$

(ii) $x + 7y + 2c + 2d + 3c + 3d + 6x + 6y + 10x$.

Exercises 9b.

1. Explain graphically the meaning of the following :—

(i) $a - 2b - c$.

(ii) $2a - b + c$.

(iii) $-4a + 4b + 4c$. Take as values, $a = \frac{1}{2}''$, $b = \frac{1}{4}''$, $c = 1''$.

2. Subtract and give the numerical value of the following :—

(i) $\begin{array}{r} 5a + 6b - c \\ 2a - 3b + c \\ \hline \end{array}$ (ii) $\begin{array}{r} 10a + b + c \\ 9a + 7b - 6c \\ \hline \end{array}$ (iii) $\begin{array}{r} a + 12b + 16c \\ a + 10b + 16c \\ \hline \end{array}$

Take as values, $a = \frac{1}{2}''$, $b = \frac{1}{4}''$ $c = 1''$.

3. Arrange like quantities in columns and find the value of :—

(i) $7x - 5y + 3 - 4 + 8y - 10x + 3x + 2y + 1$.

(ii) $5W - 5X + 5Z - W - 6X - 2Z - 6W + X - Z$.

4. Explain graphically the meaning of :—

$a - 2b + 3a - b$. Let $a = \frac{1}{2}''$ and $b = 1''$.

5. Subtract :—

(i) $x^2 + 7xy + y^2$ from $0{\cdot}5x^2 + 3xy - 2y^2$.

(ii) $10x^3 - 5Q^2 + p$ from $3x^3 - 8Q^2 - 7p$.

6. Subtract $2a + 3b - c$ from $a - 4b + 5c$ and also $x + 6y - 2a$ from $a - 5x + 7y$. In the latter portion of this exercise prove your answer by substituting the following values :—$x = 0{\cdot}1''$, $y = 0{\cdot}5''$ $a = 0{\cdot}75$."

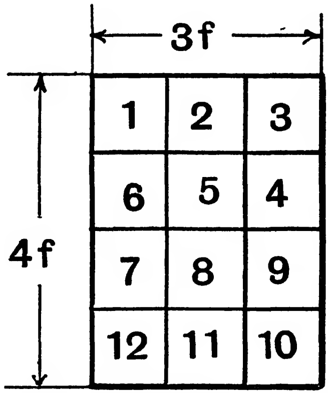

Fig. 48.

Algebraic Multiplication.—Fig. 48 shows a rectangle 3 units long and 4 units wide. To get the area of such a figure we multiply 3 by 4 and get 12 units.

But the new unit is different from the original unit, being in what is called "square measure." This new unit is the area of a square whose edge is 1 unit long. Twelve such squares are shown in Fig. 48. Now suppose the unit of length is 1 foot and represent this amount by the symbol f. The length of the rectangle is now $3f$ and its width $4f$. The area is now $3f \times 4f$ which equals 12 new units, the new unit being the area of a square of edge f, which might be expressed by $f \times f$. For convenience we write this f^2 and call it "f squared." The area of the rectangle in Fig. 48 is therefore $12f^2$.

In this and the following case the student must remember that the diagrams are only *illustrations* of algebraical laws. He should never lose sight of the fact that in algebra f (or any other letter) can only represent a *number*. The above paragraph referring to Fig. 48 should now be worked through again, letting f represent a number, say 3. The diagram should be drawn again, dividing each distance which in Fig. 48 represents f into three equal parts. This will have the effect of dividing each of the twelve squares into 3^2 ($= 9$) smaller squares. The diagram should now contain $12 \times 3^2 = 108$ little squares. Does it?

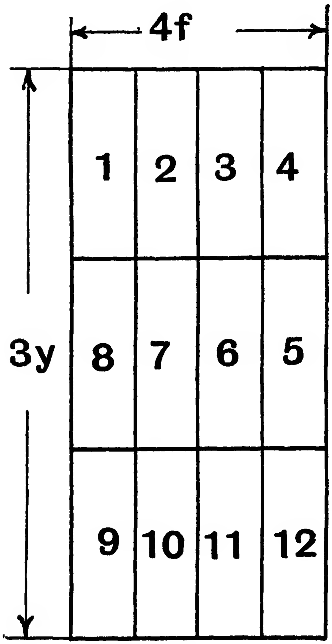

Fig. 49.

In Fig. 49 is shown a rectangle whose length is 4 feet ($4f$) and whose width is 3 yards ($3y$). The area of such a rectangle is $4f \times 3y = 12fy$. We see that the area is again 12 units, but the unit is not a square foot (f^2) nor a square yard (y^2) but what may be called a "foot-yard" (fy) being itself a rectangle 1 foot by 1 yard.

In just the same way the volume of a rectangular solid 5 feet ($5f$) long, 4 feet ($4f$) wide, and 2 feet ($2f$) deep would be $5f \times 4f \times 2f = 40f^3$, which is read "$40f$ cubed." The unit (f^3) in this case is the volume of a cube whose edge is 1 foot (f) long. Again, if the solid had been 5 yards ($5y$) long instead of 5 feet, the other dimensions remaining the same, the volume would be given by $5y \times 4f \times 2f = 40\ yf^2$. It is again 40 units, but the new unit might be called a "yard-square-foot," being the volume of a rectangular solid 1 yard long, 1 foot wide and 1 foot thick.

In an expression such as f^3, the quantity 3 is called the index of f. It is important to remember that $f^3 = f \times f \times f$. We sometimes meet with expressions like a^5 and $x^4\ y^3$. These are read "a to the fifth" and "x to the fourth y cubed." They are equal respectively to

$$a \times a \times a \times a \times a \text{ and } x \times x \times x \times x \times y \times y \times y.$$

No geometrical meaning can be given to such expressions as these. When a symbol has no written index, as $3a$, we understand the index of a to be 1 (*not* 0).

We are now in a position to multiply two algebraic quantities together. Take $3a^2b \times 5ab$. This may be expressed as $3 \times a \times a \times b \times 5 \times a \times b$. Collecting the like quantities together we have $(3 \times 5) \times (a \times a \times a) \times (b \times b)$ and this is equal to $15a^3b^2$.

Here then is the

Rule for Algebraic Multiplication.—*Multiply the coefficients and* **add** *the indices of the like quantities.*

Thus to obtain the product of $3x^2y$ and $7x^3y^4$ we get (3×7) $(x^{2+3})\ (y^{1+4}) = 21x^5y^5$.

So far we have given no thought to the signs of the expressions whose products are required. Collecting from our foregoing experience we obtain the following results:—

$$\begin{aligned} +a \times +b &= +(+ab) = +ab \\ +a \times -b &= +(-ab) = -ab \\ -a \times +b &= -(+ab) = -ab \\ -a \times -b &= -(-ab) = +ab \end{aligned}$$

From these results we get a

Rule of Signs.—*In the multiplication of two quantities, two like signs give a plus and two unlike signs a minus.*

Long Multiplication.—We will now consider one or two examples in multiplication. Suppose that it is required to evaluate :—

$$(a+b)^2, \text{ that is } (a+b) \times (a+b)$$

$$\begin{array}{lr} & a + b \\ & \underline{a + b} \\ (a+b) \times a \text{ ....} & a^2 + ab \qquad \\ (a+b) \times b \text{ ....} & \underline{ab + b^2} \\ \text{Sum ....} & \underline{a^2 + 2\,ab + b^2} \end{array}$$

The mode of procedure is to set the two expressions down as shown, then to multiply the whole of the expression in the top line by each term in the multiplier separately, setting out the products so that only like terms fall in the same column; finally the sum of the products is taken.

It forms a useful check on the work to substitute any values taken at random into the two expressions thus :—Let $a = 2$ and $b = 3$. Then $(a + b) = 5$ and $(5)^2 = 25$. That is, the product $a^2 + 2\,ab + b^2$ should be equal to 25 when a is taken as 2 and b as 3.

$$\begin{array}{ll} \text{Substituting ....} & a^2 + 2ab + b^2 \\ & = (2)^2 + (2 \times 2 \times 3) + (3)^2 \\ & = 4 + 12 + 9 \\ & = 25. \end{array}$$

Many simple products may be represented geometrically. In Fig. 50, p. 83, $(a + b)$ is represented by the straight line PQ. Then the square on PQ represents $(a + b)^2$. The diagram shows clearly that this is equal to $a^2 + 2\,ab + b^2$.

Suppose $(x + y)^3$ is required. This is equal to

$$(x+y) \times (x+y) \times (x+y)$$

which is the same as $(x + y)^2 \times (x + y)$. From the last example we know that $(x + y)^2 = x^2 + 2xy + y^2$. So the problem is set out thus :—

$$\begin{array}{lr} & x^2 + 2xy + y^2 \qquad\qquad \\ & \underline{x + y \qquad\qquad} \\ (x^2 + 2xy + y^2) \times x \text{ ....} & x^3 + 2x^2y + xy^2 \qquad \\ (x^2 + 2xy + y^2) \times y \text{ ....} & \underline{x^2y + 2xy^2 + y^3} \\ \text{Sum ....} & \underline{x^3 + 3x^2y + 3xy^2 + y^3} \end{array}$$

The student should test the accuracy of this product by substituting into it any values for x and y which he cares to choose. The geometrical interpretation of this expression is possible, but it makes a somewhat complicated drawing as we are here dealing with a solid.

Ex. 43.

	$3p^2 - 2pq + 4q^2$
	$4p - 2q$
$(3p^2 - 2pq + 4q^2) \times 4p$	$12p^3 - 8p^2q + 16pq^2$
$(3p^2 - 2pq + 4q^2) \times -2q$	$- 6p^2q + 4pq^2 - 8q^3$
Sum	$12p^3 - 14p^2q + 20pq^2 - 8q^3$

Note the application of the rule of signs.

Exercises 9c.

Find the product of :—

1. $3x$ and $4b$. **2.** $3xy$ and $5x$. **3.** $6pq$ and $2p$.

4. $15ab$ and $-2b$. **5.** $4xy$ and $-xy$. **6.** $-abc$ and $-2ac$.

7. $2a - 3$ and $a + 4$. **8.** $x - 8$ and $x + 7$.

9. $2x - 4$ and $2x + 4$. **10.** $x^2 - 4x + 2$ and $3x$.

11. $2a^2 - 4ab + 2b^2$ and $a + 2b$.

12. $5p^2 - 2pq + 3q^2$ and $2p - q$.

Algebraic Division.—Consider the division of any number, say 56 by 7. We might represent the operation in the form of a fraction, thus :—$\frac{56}{7}$. Now the factors of 56 are 7 and 8, so the fraction becomes $\frac{\cancel{7} \times 8}{\cancel{7}}$. The 7's cancel and we have the result, viz. 8.

Now exactly the same process may be employed in algebraical division. Thus :—$6ab \div 2a$ becomes $\frac{\overset{3}{\cancel{6}}\cancel{a}b}{\cancel{2}\cancel{a}} = 3b$.

Again $\qquad 15a^3b^2 \div 5ab^2 = \frac{15a^3b^2}{5ab^2},$

$$= \frac{3 \times \cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{\cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b}} = 3 \times a \times a = 3a^2.$$

The quantity which is undergoing division is called the *dividend*, while the result is known as the *quotient*. Thus :—

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient.}$$

We can now state the

Rule for Algebraic Division.—*Divide the coefficients and subtract the indices of the like quantities.*

It should be noticed in the foregoing example that, applying this rule rigidly, we obtain $\frac{15}{5}a^{3-1}\,b^{2-2} = 3a^2b^0$. Now b^0 is not the same as b, for the latter is really b^1. b^0 in the above expression is the result of $\frac{b^2}{b^2}$ which clearly is equal to 1. Hence the expression is written $3a^2$.

The **Rule of Signs for Division** *is obviously the same as that for Multiplication.* Thus :—

$$\frac{ab}{a} = \frac{+\cancel{a} \times +b}{+\cancel{a}} = +b$$

$$\frac{-ab}{a} = \frac{+\cancel{a} \times -b}{+\cancel{a}} = -b$$

$$\frac{ab}{-a} = \frac{-\cancel{a} \times -b}{-\cancel{a}} = -b$$

$$\frac{-ab}{-a} = \frac{-\cancel{a} \times +b}{-\cancel{a}} = +b.$$

Long Division.—Let us now consider the problem of dividing such an expression as $15x^2 - xy - 6y^2$ by $3x - 2y$. It should be set out thus :— $3x - 2y)15x^2 - xy - 6y^2($

First see that both expressions are in order, *i.e.* that the highest power of x (the expression containing x with the highest index) comes first and the lower powers of x follow in order. Next divide the first term of the dividend by the first term of the divisor ($15x^2 \div 3x = 5x$) and place the result in the quotient thus :— $3x - 2y)15x^2 - xy - 6y^2(5x + 3y$

Multiply the *whole* of the divisor by $5x$ } .... $\underline{15x^2 - 10xy}$

Subtract and, if it is wanted, bring down the next term from the dividend .... $9xy - 6y^2$

Repeat the first process .... $9xy - 6y^2$

Subtract ....

The result is therefore $5x + 3y$.

Check the result by substituting any values for x and y, say $x = 1$ and $y = 2$. Thus :—

$$\frac{15x^2 - xy - 6y^2}{3x - 2y} = \frac{(15 \times 1) - (1 \times 2) - (6 \times 4)}{(3 \times 1) - (2 \times 2))} = \frac{15 - 26}{3 - 4} = \frac{-11}{-1} = +11.$$

That is, $5x + 3y$ should be equal to 11 when $x = 1$ and $y = 2$.

Substituting :—

$$\begin{aligned} & 5x + 3y \\ &= (5 \times 1) + (3 \times 2) \\ &= 5 + 6 \\ &= 11 \end{aligned}$$

Factors.—The following factors are very important. They should be verified by the student and then committed to memory.

$$\begin{aligned} (a + b)^2 &= a^2 + 2ab + b^2 \\ (a - b)^2 &= a^2 - 2ab + b^2 \\ (a + b)(a - b) &= a^2 - b^2 \\ (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\ (a - b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \\ (a + b)(a^2 - ab + b^2) &= a^3 + b^3 \\ (a - b)(a^2 + ab + b^2) &= a^3 - b^3. \end{aligned}$$

NOTE CAREFULLY.—There are *no* factors of $(a^2 + b^2)$.

The symbols a and b are used throughout. The student will of course understand that any other letters might be used, or a concrete quantity might be met with. Thus $a^2 - 16$ might be regarded as $a^2 - (4)^2$ the factors of which are $(a + 4)$ $(a - 4)$. Again $a^3 + 8 = a^3 + (2)^3$ and the factors are $(a+2)$ $(a^2 - 2a + 4)$.

Exercises 9d.

Divide the following :—

1. $15xy^2$ by $3y$. **2.** $12x^2y$ by $3y$.

3. $27pqr^2$ by $9pr$. **4.** $x^2 - 5x + 6$ by $x - 3$.

5. $x^2 + 7x + 12$ by $x + 4$. **6.** $3x^2 + 13x - 10$ by $x + 5$.

Write down values for the following :—

7. $(p + 2q)^2$. **8.** $(a - 3b)^2$. **9.** $(2x - y)^2$.

These products should be represented graphically ; any suitable length may be used to represent the letters. Divide the square into its component parts and mark the area of each as is done in Fig. 50.

10. $(a + y)^3$. **11.** $(2p - q)^3$. **12.** $(a - 2b)^3$.

Factorise the following :—

13. $x^2 - 64$. **14.** $a^2 - 6a + 9$. **15.** $x^2 + 8a + 16$.

16. $x^3 - 8$. **17.** $a^3 + 27$. **18.** $64 - p^3$.

Exercises 9e.

1. Explain diagrammatically the meaning of the following :—

(i) $a + b - c$.

(ii) $a + 3b - 2c$.

(iii) $2a - (2b + 3c)$.

Take as the value of $a = \frac{1}{2}''$, $b = \frac{1}{4}''$, $c = \frac{3}{4}''$.

2. Add together

$$\begin{array}{r} a - 3b + 6c \\ a + b - 2c \\ 2a - 3b + c \end{array}$$

Verify your answer by substituting the following values of a, b, and c, $a = 1''$, $b = 0{\cdot}5''$, and $c = 2''$.

3. Add together

$$\begin{array}{r} a - \phantom{1{\cdot}5}b + x \\ 2a + 1{\cdot}5b - 3x \\ 3a - \phantom{1{\cdot}}2b - 4x \end{array}$$

Verify your answer by inserting the following values $a = 2''$, $b = 3''$, and $x = 4''$.

4. A man buys z files + $2w$ screwdrivers + x cold chisels; the following day he buys $2z$ files + $6w$ screwdrivers + $5x$ cold chisels; on a third day he buys $3z$ files + $8w$ screwdrivers + $2x$ cold chisels. How many of the various tools does he buy, supposing that $z = 3$, $w = 5$, and $x = 4$?

5. Three circles are to be drawn, one of radius of w inches, another b inches and a third of y inches. State the total area algebraically as simply as possible and find the total area when $w = 1''$, $b = \frac{1}{2}''$, $z = \frac{1}{4}''$.

6. Simplify (i) $\dfrac{P.l.n}{33000} + \dfrac{q.l.n}{33000}$.

(ii) $0{\cdot}64 \times 0{\cdot}97 \times A \times \sqrt{(2gh)} + 0{\cdot}57 \times 0{\cdot}85 \times A\sqrt{(2gh)}$.

7. Add together

$$\begin{array}{rrr} \pi a^2 + & \pi b^2 + & \pi c^2 \\ 2\pi a^2 + & 3\pi b^2 - & 6\pi c^2 \\ -3\pi a^2 - & 2\pi b^2 - & 3\pi c^2. \end{array}$$

Subtraction.

8. Explain graphically the meaning of $(a - 2b) - (3a - b)$. Let $a = \frac{1}{2}''$, $b = 1''$.

9. Subtract $2a + 3b - c$ from $a - 4b + 5c$
and $x + 6y - 2a$ from $a - (5x - 7y)$.

In the latter portion of this example prove your answer by substituting the following values: $x = 0{\cdot}1''$, $y = 0{\cdot}5''$, $a = 0{\cdot}75''$.

10. Subtract

$$ax + 8bx + 3ab \text{ from } 3ax - (2bx - 2ab)$$

and if $a = 1''$, $b = \frac{1}{2}''$, and $x = \frac{3}{4}''$ draw the rectangles represented on squared paper. Show that the final area thus obtained is equal to the area as determined by the substitution of the above values.

11. Simplify

$$\left\{\frac{\pi D^2}{4} - \frac{\pi d^2}{4}\right\} - \left\{\frac{3\pi D^2}{2} - \frac{6\pi d^2}{2}\right\}$$

and also

$$(Qa + Qb) - (Pa + aQ) + (3Qa - 3Qb) - (2Qa - 2Pa).$$

12. Subtract

$$x^2 + 7xy + y^2 \text{ from } 0{\cdot}5x^2 + 3xy - 2y^2$$

and also $$10x^3 - 5Q^2 + p \text{ from } 3x^3 - 8Q^2 - 7p.$$

13. If a, b, c, and d represent the sides of rectangles, find the total area of the following:—

$$\begin{array}{rrrr} 5ab + & cd + & 5cb + & 2ad \\ -4ab - & 3cd + & cb - & 2ad. \end{array}$$

By substituting values $a = \frac{1}{2}''$, $b = 2''$, $c = \frac{1}{4}''$, and $d = \frac{3}{4}''$ give the numerical value of the final area of the rectangles.

Multiplication.

14. Multiply $(x + 3y + z)$ by $6a$ and also $x^2 + 3xy + xz$ by $3x$.

15. The base of a triangle is $10x$ units long and its height is $5x$ units. Determine the area, and if $x = \frac{1}{2}''$ give the numerical value.

16. Multiply

$$(x + 2p) \text{ by } (x + 2p)$$
$$(x + 3p) \text{ by } (2x + 3p)$$
$$(x^2 + 3p^2) \text{ by } (x - p)$$

and in each case if $x = 1{\cdot}5''$ and $p = \frac{3}{4}''$, give diagrams to illustrate your answers.

17. Multiply together and express in as simple a form as possible the following quantities:—

$$(9{\cdot}7 \times D \times n - 6{\cdot}44 \times D \times n + 4{\cdot}8 - 5) \text{ by } (D \times n - 1).$$

18. Multiply

$$(5 - x) \text{ by } (5{\cdot}5x - 2{\cdot}5x + 3x^2 + 4x - 6 - 2x^2).$$

Division.

19. Divide

$$\{2a + 4ab - (2ab + 2ab) + 8b\} \textbf{ by } a + 4b$$
$$\{4a^2 + 4ab + b^2\} \text{ by } 2a + b$$
$$x^2 - y^2 \text{ by } (x - y).$$

20. Simplify each of the following expressions :—

$$\frac{\pi \, . \, r^2 \, . \, p}{2\pi \, . \, r \, . \, t} \qquad \frac{\frac{1}{2}(a^2 - b^2)}{a - b} \qquad \frac{\frac{3}{4}(a^3 - b^3)}{\frac{1}{4}(a^2 - b^4)}.$$

21. Divide $a^2 - b^2 + 2ab + 5b^2 + 4ab + 4b^2$ by $a + 4b$.

22. Express as simply as possible

$$\frac{\pi (D^3 - d^3)}{32 (D - d)},$$

and also

$$\left(\frac{Wl}{2} + \frac{Wl}{8} + \frac{Wl}{16}\right) \div \frac{W}{2}.$$

23. Determine the value of $\dfrac{bH^3 + Bh^3}{6H(Bh + b)}$

when $B = 2$, $h = 1$, and $b = \frac{1}{2}$ in its simplest form.

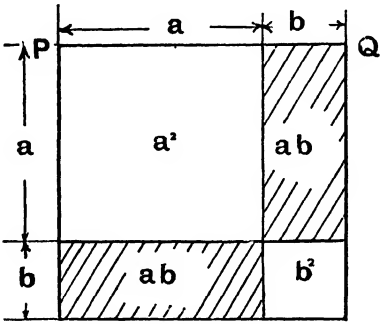

Fig. 50. (See p. 76.)

24. Divide $x^2 + 2ax - b^2 - 2ab$ by $x + 2a + b$

$3x^2 - 4x + 1$ by $3x - 3$

and also $a^2 + 3b^2 - c^2 + 2bc - 4ab$ by $a - 3b + c$

CHAPTER X.

SIMPLE EQUATIONS.

In Chapter VIII. we spoke of the expression $A = \pi r^2$ as an equation, and when we had substituted numerical values for π and r^2 and simplified the expression we got a value for A.

Let us think of a simple case. Suppose $5x = 10$. Here x is the **unknown**, as it is called, and we are required to find the value for it. Dividing both sides of the equation by the coefficient of x (viz. 5), we obtain $x = \frac{10}{5} = 2$. So that 2 is the solution required.

This operation may be spoken of as **solving an equation,** and the value for x which we obtained would be called the **solution**.

The methods of dealing with various types of equations will be gathered from the following examples:—

Ex. 44. *Solve the equation*:— $\dfrac{x}{7} = 3$.

Multiply both sides of the equation by 7 ... $x = 7 \times 3 = 21$.

The student should always substitute the value he has found for the unknown and see if it "satisfies" the equation. Thus, putting 21 instead of x in the above equation we get $\dfrac{21}{7} = 3$. Is this true?

Ex. 45. *Solve the equation*:— $2x - 4 = 3x - 7$.

Adding	$3x$ to both sides	$2x - 3x - 4 = -7$
Adding	4 to both sides	$2x - 3x = -7 + 4$
Therefore		$-x = -3$
Therefore		$+x = +3$

The student should notice that "adding $-3x$ to both sides" of the equation had the effect of moving the $+3x$ from the right hand to the left hand side of the equation and changing its sign from + to −, whilst "adding + 4 to both sides" had

the effect of moving the -4 from the left hand to the right hand side of the equation and changing its sign. Hence follows a rule:—**In an equation any expression may be moved from one side of the equation to the other, with its sign changed.**

Ex. 46. *Solve the equation* :— $\frac{x}{4} = 1\frac{5}{16} + \frac{3}{8}$.

Converting the mixed number $\qquad \frac{x}{4} = \frac{21}{16} + \frac{3}{8}$

Bringing every term to the same common denominator (16) $\qquad \frac{4x}{16} = \frac{21}{16} + \frac{6}{16}$

Multiplying every term by 16 $\qquad 4x = 21 + 6$

Therefore $\qquad 4x = 27$

Dividing both sides by 4 $\qquad x = \frac{27}{4} = 6\frac{3}{4}$.

Exercises 10a.

Solve the equations :—

1. $\frac{2x}{5} = 1\frac{2}{3}$. **2.** $\frac{3}{x} = \frac{1}{5}$. **3.** $2d - 3 = 5d - 6$.

4. $\frac{2a}{3} + 1 = 4a - 7$. **5.** $\frac{b}{2} = b - 1\frac{3}{4}$. **6.** $\frac{5}{h} = \frac{2}{h} + 1\frac{1}{2}$.

7. $3(R - 4) = 2(2R - 7)$. **8.** $\frac{x}{3} - 2 = x - 6\frac{1}{2}$.

9. $4\frac{1}{2}(2b - 6) = \frac{3}{4}(8b - 6)$.

Ex. 47. *If a piece of wire has an electrical resistance of R ohms, and an electric current is passed through it so that its extremities have a difference of potential of E volts, then the current (call it C ampères) which flows through the wire is given by the formula* $C = \frac{E}{R}$. *What potential difference is required to send a current of 4 amps. through a wire having a resistance of 7 ohms?*

The formula is $\qquad C = \frac{E}{R}$

Substituting the values given $\qquad 4 = \frac{E}{7}$

This is the equation to be solved, and E is the unknown.

Multiply both sides by 7 $\qquad 28 = E$

Therefore 28 volts would be required.

With the same voltage, what resistance will be required for the current to be three ampères only?

Formula	$C = \dfrac{E}{R}$
Substituting the values given	$3 = \dfrac{28}{R}$
(R is the unknown)	
Multiplying both sides by R	$3R = 28$
Dividing both sides by 3	$R = \frac{28}{3} = 9\frac{1}{3}$

Required resistance, $9\frac{1}{3}$ ohms.

Ex. 48. *During recent years electricity has been largely used for heating purposes. If H represents the number of heat units (calories they are called) produced by a current of C ampères flowing through a resistance of R ohms for t seconds we have the formula—$H = 0{\cdot}24\ C^2 . R . t$. Find the current required to produce* 1,000 *calories per minute in a resistance of* 20 *ohms.*

Formula	$H = 0{\cdot}24 . C^2 . R . t$
Substituting the values given	$1000 = 0{\cdot}24 \times C^2 \times 20 \times 60$
Therefore	$1000 = 288\ C^2$
(Here C is the unknown)	
Dividing both sides by 288	$\frac{1000}{288} = C^2$
Therefore	$C^2 = 3{\cdot}47$
Therefore	$C = \sqrt[2]{3{\cdot}47} = 1{\cdot}46$ (number of ampères necessary)

The student should now find what voltage would be required to send this current through the resistance of

$$20 \text{ ohms. } \left(C = \frac{E}{R}\right).$$

Ex. 49. *If the coil of wire conveying the current in the previous example were placed in a copper vessel weighing w grams and containing W grams of water at $t°$ C., and H calories of heat raised their temperature to $T°$ C., then $H = (W + 0{\cdot}09w)(T - t)$. Find how hot the water would become in* 1 *minute if the copper vessel weighed* 38 *grams and contained* 100 *grams of water at* 15° C.

Formula	$H = (W + 0{\cdot}09w) . (T - t)$
Substituting the values given	$1000 = (100 + 0{\cdot}09 \times 38) (T - 15)$
Therefore	$1000 = (103{\cdot}4) (T - 15)$
(Noting that T is the unknown)	
Multiplying out	$1000 = (103{\cdot}4\ T - 1550)$
Adding 155 to both sides	$2550 = 103{\cdot}4\ T$
Dividing both sides by 103·4	$\frac{2550}{103{\cdot}4} = T$
Therefore	$T = 24{\cdot}6°\ \text{C}.$

Ex. 50. *If wrought iron piping has an external diameter of D inches, and an internal diameter of d inches, then the weight (W lb.) of a length of l″ is given by:—*

$$W = 0{\cdot}069\ l\pi\ (D^2 - d^2).$$

If one foot of such piping of 1″ *internal diameter weighs* $3\frac{1}{4}$ *lb., find its external diameter.*

Formula	$W = 0{\cdot}069 . l . \pi . (D^2 - d^2)$
Substituting the values given	$3{\cdot}25 = 0{\cdot}069 \times 12 \times 3{\cdot}1416 \times (D^2 - 1^2)$
(D is the unknown)	
Multiplying out	$3{\cdot}25 = 2{\cdot}6\ (D^2 - 1)$
Removing the bracket	$3{\cdot}26 = 2{\cdot}6\ D^2 - 2{\cdot}6$
Adding 2·6 to both sides	$5{\cdot}86 = 2{\cdot}6\ D^2$
Dividing both sides by 2·6	$\frac{5{\cdot}86}{2{\cdot}6} = D^2$ ...... $D^2 = 2{\cdot}26$

Therefore D $= \sqrt{2{\cdot}26} = 1{\cdot}5''$ *i.e.* the external diameter is $1\frac{1}{2}''$.

Exercises 10b.

The student may refer back to the text of the chapter for the formulae required in some of the following problems.

1. What voltage is required to pass a current of 1·75 amps. through a resistance of 12 ohms ?

2. A potential difference of 15 volts sends a current of 2·6 amps. through a coil of wire. What is the resistance of the wire ?

3. Find the current required to generate 750 calories of heat per minute in a resistance of $18\frac{1}{2}$ ohms.

4. If the coil carrying the current in Problem 3 were contained in a copper vessel weighing 38 grams and containing 150 grams of water at 12° C., what would be the temperature of the water after 2 minutes?

5. If a wrought iron pipe has an internal diameter of $\frac{3}{4}''$ and an external diameter of $1\frac{1}{4}''$, what is the length of a piece of this pipe weighing $5\frac{1}{2}$ lb.?

6. Find the diameter of circles whose areas are respectively 2 sq. in., 5·6 sq. in. and 16·7 sq. in.

7. If a body is allowed to fall from rest under the action of gravity, its velocity (v ft. per sec.) after it has fallen s ft. is given by $v^2 = 2g\,s$ (where $g = 32{\cdot}2$). Find the distance a body must fall in order to acquire a velocity of 100 ft. per second.

8. If a rod of metal has a length of l cm. at 0° C., and at °t C. its length becomes L, then $L = l\,(\mathrm{I} + at)$. Where a is a quantity depending on the metal it is called the coefficient of linear expansion and for brass is 0·000018. If a brass rod is 316 cm. long at 0° C., at what temperature will it be 317 cm.?

9. A number of forces, some being pulls and others pushes, act on a point and in the same straight line. On one side the following forces act: $5x$ lb. + 6 lb. + 7 lb. − $2x$ lb. +$3x$ lb., and these are just equal to the forces on the other side, which are $2x$ lb. + 10 lb. + $5x$ lb. − $3x$ lb. + 12 lb. Determine the value of x lb.

10. A tendency to twist a body is measured in "lb.-feet," and the following values represent the tendency to twist in the same direction as the hands of a clock move: $5\,B$ lb.-ft. + 21 lb.-ft. + $3\,B$ lb.-ft + 10·5 lb.-ft. These are just balanced by $10\,B$ lb.-ft. acting in the opposite direction. Determine the value of B ft.

11. Taking the above example, suppose the tendency to twist is: $5\,W$ lb.-ft. + 14·9 lb.-ft. + $10\,W$ lb.-ft. + 5 lb.-ft., and this

is just balanced by 10 W lb.-ft. + 26 lb.-ft., what is the value of W lb?

12. The following tendencies to twist a body: 2 tons × 2 ft. + 3 tons × 4 ft. + 5 tons × 6 ft. + S tons × 8 ft. are just balanced by S tons × 10 ft. Determine the value of S.

13. If wrought iron will safely carry 5 tons per sq. in. in tension, find the area necessary to carry 2 tons. Use this equation to find the result:

$$\frac{\text{load}}{\text{area}} = 5 \text{ tons sq. in.}$$

Also if the area of 1·4 sq. in. is available, determine the load it will carry.

14. If the pressure per sq. ft. of water (P) = 62·4 × head of water in feet, (H), determine the pressure per sq. ft. for a head of water of 1 ft. and also of 1 inch, and also the head equivalent to 1 lb. per sq. in.

15. We are told in books of applied mechanics that $\frac{\text{stress}}{\text{strain}} = E$, where $E =$ the modulus of elasticity, stress = load per sq. in. and strain alteration of unit length under stress. If the stress $= \frac{1300 \text{ lb.}}{\cdot 0491 \text{ sq. in.}}$ and strain $= \frac{0{\cdot}007''}{8''}$ determine E. Also if $E = 30{,}200{,}000$ lb. per sq. in. and the stress $\frac{1400 \text{ lb.}}{0{\cdot}0491 \text{ sq. in.}}$ determine the strain.

16. The theoretical velocity of water due to a given head in feet is found from the following:—$V = \sqrt{64{\cdot}4 \times H}$, where $H =$ head of water in feet, and $V =$ velocity in feet per second. Determine H when V is:—10, 15, 18, and 20 ft. per second respectively.

17. Using the equation $\frac{\text{load}}{\text{area}} =$ stress (5 tons per sq. in.), determine the diameter of round bars to carry loads of 8, 10, 12, and 24 tons respectively.

18. Determine the pitch of a riveted joint from the formula—
Efficiency per cent. $= \frac{(p - d)t}{p \times t}$ where $t = \frac{3}{8}''$, $d = \frac{3}{4}''$, efficiency per cent. $= 60$ per cent. (*i.e.* $\frac{60}{100}$), and $p =$ pitch of the rivets.

19. The following law was determined for a piece of machinery:—Friction $= 0 \cdot 2 + 0 \cdot 0112 \times$ load in lb. Determine the load when the friction is equal to 0·38, 0·4, 0·42, and 0·45 lb. respectively.

20. In testing a worm and worm wheel the following relation was found experimentally:—Friction in lb. $= 0 \cdot 048 + 0 \cdot 023 \times$ load in lb. Determine the load when the friction was 0·47, 0·62, and 1·1 lb. respectively.

21. A formula used in the design of struts is:—

$$p \text{ (stress)} = 42{,}000 - 128 \frac{l}{r}.$$

Determine the different ratios of $\frac{l}{r}$ when $p = 16{,}000, 20{,}000$, and 24,000 respectively.

22. A close paling fence is to be supported by 3″ by 4″ oak posts spaced 9 ft. apart. Determine how many posts must be ordered for a plot of ground which is rectangular in shape 200 ft. by 25 ft., the fence to completely enclose the ground.

23. Railway sleepers 10″ by 5″ in cross-section are laid along a mile of railway track. If 2112 are used per mile, what is the distance from centre to centre of each sleeper.

24. Grindstones should run at about 800 ft. per min. (peripheral velocity). Determine the revolutions per min. a 3 ft. dia. grindstone should make. Circular saws for cutting wood should run at about 9,000 ft. per min. Determine how many revs. per min. saws of 26″, 44″, and 50″ dia. should make. The rule being:—Peripheral velocity (rim or outer surface velocity) = revs. per minute × circumference of the rotating body in feet.

25. Determine the length of belt required for the two pulleys shown in Fig. 51. The rule for calculating the length of belt for

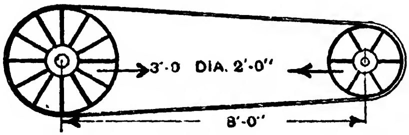

Fig. 51.

an open drive (neglecting the joint) being :—Add together the circumference of both driven and driving pulleys, divide this by 2, to this quantity add twice the distance between the centres of the two pulleys.

CHAPTER XI.

COMMON SOLIDS.

Surfaces and Solids.—We have seen in Chapter VI. how a plane figure is an enclosed *surface*, the latter being bounded by one or more *lines*. All surfaces possess length and breadth, but never thickness; we speak of this as having two "dimensions." When a body has **three dimensions,** *i.e.* length, breadth, and thickness, it is called a **solid,** or a solid figure. A solid may be considered as an enclosed piece of *space*, which is bounded by one or more surfaces.

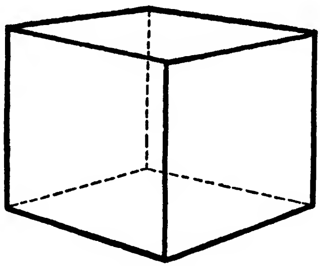

Fig. 52.

One of the simplest solids is the **Cube** which is shown in Fig. 52. The student should note that it has six sides or faces, all of which are squares. If each edge is 1″ long, then the amount of space it occupies is called one **cubic inch** which is a unit of **volume.** If the cube had an edge of 1 centimetre or 1 ft. its volume would have been 1 cubic centimetre (usually written 1 c.c.) or 1 cubic foot.

If the edge of a cube were 12″ long, it is easily seen that it might be divided up into 12 layers of 1″ cubes, each layer containing 12×12 ($= 144$) of these small cubes. In other words

the volume of such a cube is $12^3 = 1728$ cubic in. Since a cube has six faces, each of which is a square, its total surface area is six times the area of one face.

Ex. 51. *Make a cube of* 2″ *edge.*

A drawing consisting of 6 squares of 2″ edge should be made as shown in Fig. 53, preferably on stiff drawing paper. If the figure be cut out and folded about the dotted lines, a cube is obtained. The shaded portion of the diagram represent little tabs which should be left upon the figure and given a coat of gum. They then may be used for holding the model together.

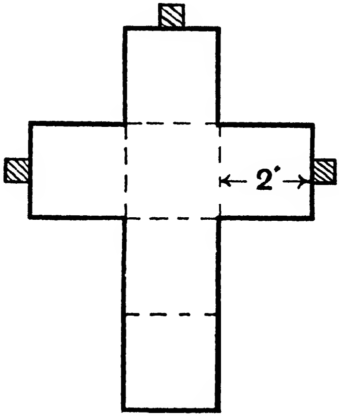

Fig. 53.

Prisms.—Fig. 54 shows a "rectangular prism" in which the length, breadth, and thickness are marked by the letters l, b, and t. It should be noted that the ends are rectangles and the sides are parallel. A little thought will show us that its volume is the product of l, b, and t. To obtain its surface area we must observe that there are two ends each having an area of bt, two sides each of area lt, and a top and bottom whose areas are lb. The sum of these areas gives the total surface area of the solid.

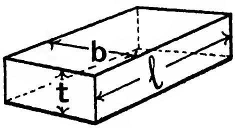

Fig. 54.

Ex. 52. *Make a rectangular prism whose length is* 3″, *breadth* 2″, *and thickness* 1″.

Fig. 55 shows the diagram. The instructions are the same as in the case of the cube.

Ex. 53. *Make a triangular prism of length* 8 *cm. The triangular face of which is an isosceles triangle whose equal sides are* 4·6 *cm. and whose other side is* 3 *cm.*

Fig. 56 shows the triangular prism and Fig. 57, the diagram for making it. It should be noticed that the total surface area is the sum of that of the two triangles and the three rectangles shown in Fig. 57. The volume is the product of the area of the triangular end and the length.

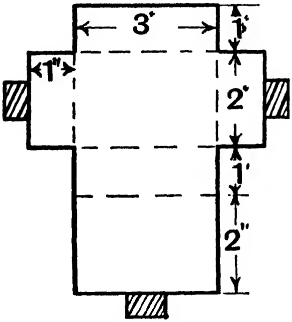

Fig. 55.

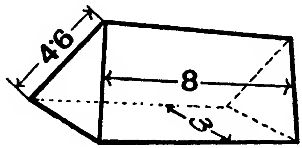

Fig. 56.

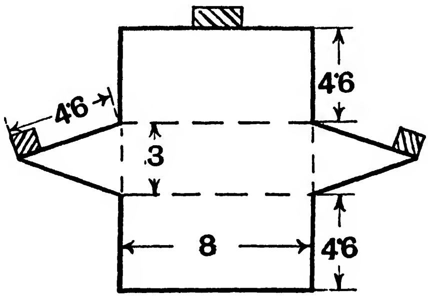

Fig. 57.

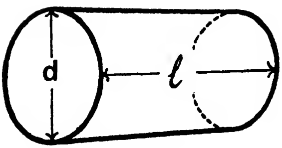

Fig. 58.

The Cylinder.—Closely allied to the prism is the cylinder, which differs in that it has circular ends. Fig. 58 shows a cylinder of diameter d and length l. The surface area should be considered in two portions:—(1) the flat surfaces, which are the two circular ends, and (2) the curved surface. The latter, if it could be "peeled off" and laid out flat, would

be a rectangle of length l and whose breadth is equal to the circumference of the circular end, which is πd. The curved surface is therefore $\pi \times dl$. The volume is the product of the area of the circular end and the length, and is $0{\cdot}7854\, d^2 l$.

Ex. 54. *Make a cylinder of length 3″ and diameter of $1\frac{1}{2}$″.*

The diagram is shown in Fig. 59.

The Cone.—A cone is shown in Fig 60. It has a circular base (of diameter d) and a curved surface terminating at a point called the *apex*. If the line joining the apex to the centre of the base

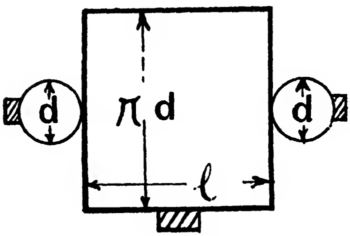

Fig. 59.

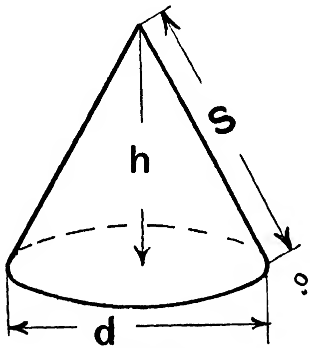

Fig. 60.

is perpendicular to all diameters of the base, the solid is called a **right cone.** The length of this line may be called the vertical height, and is denoted in the figure by h. The length of the slanting side is called the slant height and is denoted in the figure by s. If the radius of the base is called r, we can see that a right-angled triangle is formed by these three lines and consequently we have the relation $s^2 = h^2 + r^2$.

Fig. 61 shows the surface of the cone laid out flat. The curved surface forms part of a circle whose whole circumference would be $2\pi s$, the length of a portion of this circumference in the figure being πd. Consequently its area will be $\frac{\pi d}{2\pi s}$ of the area of the whole circle (*i.e.* of πs^2).

Therefore the curved surface $= \dfrac{\pi d}{2\pi s} \times \pi s^2$.

Cancelling we have $= \frac{1}{2}\pi ds$.

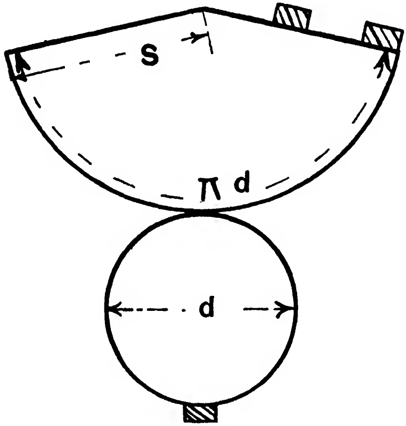

Fig. 61.

The volume of a cone is found to be one third of the product of the base and the vertical height Therefore the volume

$$= \tfrac{1}{3} \times 0\ 7854\, d^2 h.$$

Ex. 55. *Make a cone with a base of diameter 4 cm., and slant height 6 cm.*

The diagram is shown in Fig. 61.

The Sphere. — The sphere is the solid traced out by a circle rotating about one of its diameters. Everybody is familiar with the sphere as being a solid having the shape of a ball. It is impossible to lay its surface out flat and consequently a paper model cannot be made. If a sphere have a radius of r units its surface area is equal to $4\pi r^2$ and its volume to $\frac{4}{3}\pi r^3$.

Hollow Cylinders.—If a hollow "open" cylinder (*i.e.* one not having closed ends) have an external diameter of D and an internal diameter of d and a length of l, the volume V of the material of which it is composed is given by

$$V = 0{\cdot}7854.D^2.l - 0{\cdot}7854.d^2.l$$
$$= 0{\cdot}7854\, l\, (D^2 - d^2)$$

If R and r be the external and the internal radii respectively

$$V = \pi R^2 l - \pi r^2 l$$
$$\pi l\, (R^2 - r^2)$$

Hollow Sphere.—Similarly a hollow sphere having external and internal radii of R and r respectively has a volume (V) given by

$$V = \tfrac{4}{3}\pi R^3 - \tfrac{4}{3}\pi r^3$$
$$= \tfrac{4}{3}\pi\,(R^3 - r^3)$$

Collected results.

Cube of edge l.

$$\text{Total area} = 6\,l^2$$
$$\text{Volume} = l^3.$$

Rectangular prism, length l, breadth b, and thickness t.

$$\text{Total area} = 2\,(bt + bl + lt)$$
$$\text{Volume} = l\,b\,t.$$

Cylinder of diameter d (or radius r) and length l.

$$\text{Flat surface} = 2 \times 0{\cdot}7854 \times d^2 \text{ or } 2\pi r^2$$
$$\text{Curved surface} = \pi dl \text{ or } 2\pi\,rl$$
$$\text{Volume} = 0{\cdot}7854\,d^2\,l \text{ or } \pi r^2\,l.$$

Cone of radius of r or diameter d, vertical height h and slant height s,

$$s^2 = h^2 + r^2.$$
$$\text{Flat surface} = 0{\cdot}7854\,d^2 \text{ or } \pi r^2$$
$$\text{Curved surface} = \tfrac{1}{2}\,\pi ds \text{ or } \pi rs$$
$$\text{Volume} = \tfrac{1}{3} \times 0{\cdot}7854 d^2 h \text{ or } \tfrac{1}{3}\pi r^2 h.$$

Sphere of radius r.

$$\text{Surface} = 4\pi r^2$$
$$\text{Volume} = \tfrac{4}{3}\pi r^3.$$

Hollow cylinder of length l and external and internal diameters (or radii) of D and d (or R and r) respectively.

$$\text{Volume} = 0{\cdot}7854\,l\,(D^2 - d^2)$$
$$\text{or } \pi l\,(R^2 - r^2).$$

Hollow sphere of external and internal radii R and r respectively.

$$\text{Volume} = \tfrac{4}{3}\pi\,(R^3 - r^3).$$

Exercises 11a.

1. Examine the model of the cube made in Example 51. How many faces has it? How many edges has it? How many corners? What is its area, what is its volume?

2. Examine the model of the rectangular prism made in Example 52. In what ways (if any) do its faces, edges, and corners differ from those of the cube? What is its total area and volume?

3. Examine the model of the triangular prism made in Example 53. Find its total area and volume.

4. Examine the model of the cylinder made in Example 54. Find (1) the area of the flat portions of its surface, (2) the area of the curved surface, (3) its volume.

5. Examine the model of the cone made in Example 55. What is its vertical height? Find the area of its base and of its curved surface. What is its volume?

Exercises 11b.

1. How many sq. ft. of zinc No. 1. (Zinc Gauge) will be required to line a wooden tank (without lid) 3′ wide, 8′ long, and 1′ 6″ deep inside measurements? If the ordinary stock size of sheet zinc is 3′ by 8′, how many sheets must be ordered? Also determine the volume of the tank.

2. Determine the number of watts required to raise the temperature of a room 20° F. in 1 hour; the room measures 15′ 3″ by 12′ 3″ by 9′ 6″ high, and you can assume that under the best conditions 1 watt of electrical energy will raise 1 cubic ft. of air through 20° F. in 1 hour. How many heater lamps must be ordered if each lamp requires 250 watts?

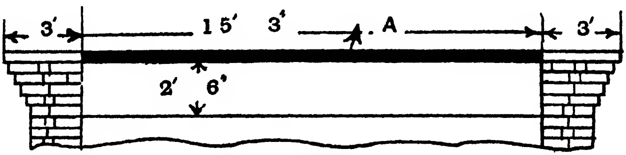

Fig. 62.

3. A railway platform is 390′ long and 21′ 3″ wide (Fig. 62). Determine the number of square feet of concrete flags required for the part *A*, and if each flag measures $2\frac{1}{2}'$ by 2′, how many must be ordered? Underneath the flags a depth of 2′ 6″ requires filling with ballast. Determine the number of cubic ft. required.

4. Galvanized wrought iron tanks are sold 2′ by 2′ by 1′ 7″ deep. What is their cubic capacity? If a cubic ft. of water measures 6·24 gallons, determine how many gallons the tank will hold. How many gallons do you think the catalogue will give?

5. A man is building a small shed whose cross section is shown in Fig. 63. If it is 15′ long, determine (*a*) the cubic feet of space inside the shed, (*b*) the number of sq. ft. of felt required to cover the roof, (*c*) its cost at 2s. a roll 15 yards by 36″ wide.

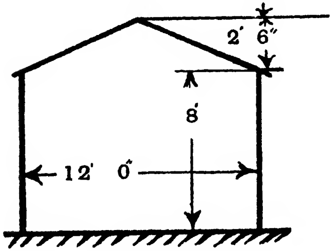

Fig. 63.

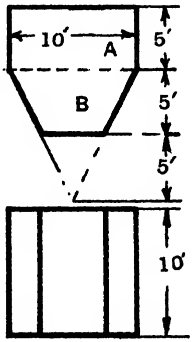

Fig. 64.

6. The end of a coal shoot is shown in Fig. 64. What is (*a*) the cubic capacity of the part *A*, (*b*) the cubic capacity of the part *B*?

7. The cylindical float of a carburettor measures 6 cm. outside diameter by 6 cm. long. Find its volume and the number of sq. cm. of copper required to make it.

8. The valve of an engine measures $\frac{3}{4}''$ diameter and its lift is $\frac{3}{16}''$ (Fig. 65). Determine the number of sq. in. in the curved part (*A*) available for the gas to flow through when the valve is fully open.

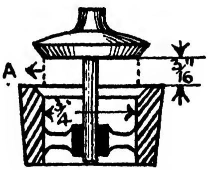

Fig. 65.

9. How many sheets of copper 4′ by 2′ must be ordered to make 15′ of 4″ diameter piping? If $\frac{1}{2}''$ lap over is required for the joint, determine the extra area required.

10. An air compressor has a cylinder whose piston is 85 millimetres in diameter and its stroke is 102 millimetres. Determine the volume swept through per stroke.

11. Fig. 66 shows part of the speed cone for a lathe. Determine, (*a*) the circumference of each speed (*i.e.*, circles 9″, 10″ and 11″ diameter), (*b*) the total volume supposing it to be solid, (*c*) the total volume of the metal (neglecting corners, curves, and shaft),

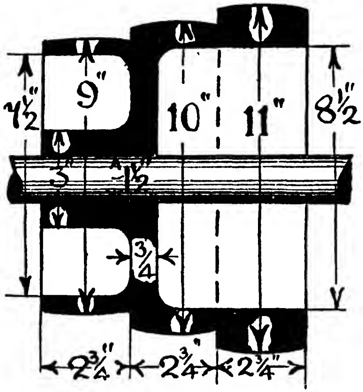

Fig. 66.

using the measurements shown in the figure. (The problem resolves itself into finding the volumes of a number of hollow cylinders.)

12. The ball on a ball float valve measures 7 cm. in diameter. Determine the volume.

CHAPTER XII.

RELATIVE DENSITIES, WEIGHTS, AND VOLUMES.

We are accustomed to speak of lead as being a heavy metal, by which we mean that a piece of lead is heavy compared with the weight of an equal volume of most other metals. Now we can express this idea numerically in two ways. We can say that lead is so many times as heavy as an equal volume of some common substance such as water, in which case the number is called the **specific gravity**, or we can give the weight of unit volume, which is called the **density.**

Lead has a specific gravity of 11·3. We never attach any units to this number, as a piece of lead is 11·3 times as heavy as the same volume of water in what ever units they are measured. The density of lead, however, may be anything according to the units employed. Thus it is 11·3 grammes per cubic centimetre, or 705 pounds per cubic foot, or 0·407 lb. per cubic inch, and so forth.

It will be noticed that the density in "grammes per cubic centimetre" is numerically the same as the specific gravity. The **gramme** is the metrical unit of weight, and may be taken as equal to the weight of one cubic centimetre of water. Thus a 1,000 c.c. (which is called a **litre**) of water weighs 1,000 grammes (which is called a **kilogramme**). It is easily seen therefore that since 1 c.c. of lead weighs 11·3 grams whilst 1 c.c. of water weighs 1 gram, any volume of lead will have a weight 11·3 times that of the same volume of water.

The following densities will be found useful:—

Material.	Density in grams per c.c. or Specific Gravity.	Density in lb. per cubic foot.	Density in lb. per cubic inch.
Water ...	1	62·4	0·036
Cast Iron ...	7·2	450	0·26
Wrought Iron	7·7	480	0·277
Mild steel ...	7·85	490	0·285
Brass... ...	8·7	542	0·313
Copper ...	8·9	555	0·32
Lead	11·3	705	0·407
Tin	7·3	456	0·263
Aluminium ...	2·6	162	0·094
Yellow Pine...	0·5	31	0·018

To determine the density of a body we must know its volume and its weight, and then we have the relation:—

$$\text{Density} = \frac{\text{Weight}}{\text{Volume}}.$$

Regarding this as an equation we may determine any one of the quantities mentioned if we know the other two. Thus

$$\text{Weight} = \text{Volume} \times \text{Density, and Volume} = \frac{\text{Weight}}{\text{Density}}.$$

These equations are of immense use in practical work for the determination of weights and volumes of materials required in certain operations. Before proceeding to the consideration of a few problems we will examine some of the units used.

In the English system a cubic foot and a cubic yard will be readily understood. The gallon, however, is a unit of volume which is frequently met with. These various units must be linked up with each other and with the Metric system, and for this purpose the following factors will be handy for reference:—

1 foot = 30·48 cm.

1 chain = 66 feet = 20 metres.

1 acre = 10 square chains = 4000 square metres.

1 cubic foot = 1728 cubic inches = 28·4 litres.

1 cubic yard = 27 cubic feet = 0·7646 cubic metres.
1 gallon = 277·3 cubic in. = 4544 c.c.
1 gallon of water weighs 10 lb.
1 cubic foot of water weighs 1000 ounces = 62·4 lb.
1 lb. = 453·6 grams.

Ex. 56. *A cube of concrete of 2′ edge weighs* 1,120 *lb. Find its density in grammes per c.c.*

Volume of the cube = 2^3 = 8 cubic feet.
Now 1 cubic ft. = 28·4 litres = 28·4 × 1,000 c.c. = 28,400 c.c.
Therefore the volume of the concrete = 28,400 × 8 = 227,200 c.c.
Also 1 lb. = 453·6 grams.
Therefore the weight of the concrete = 453·6 × 1,120 grams.

$$\text{Now density} = \frac{\text{Weight}}{\text{Volume}} = \frac{453{\cdot}6 \times 1120}{227200}$$
$$= 2{\cdot}24 \text{ grams per cubic centimetre.}$$

Exercises 12a.

1. A cubical tank has an internal edge of 2′ 6″. How many gallons will the tank hold? If the empty tank weighs 45 lb., what is its weight when full of water?

2. A gas engine cylinder has a bore of 7″ and a stroke of 11″. Express the volume traced out by the piston in c.c.

If at the back of the cylinder there is a "clearance" space of 115 cubic in, what is the total volume of the cylinder in cubic centimetres?

3. A cricket ball has a diameter of 3″ and weighs $5\frac{1}{2}$ oz. Find the mean density of the materials of which it is made: (1) in lb. per cubic ft., and (2) in grams per c.c.

4. A jam jar is filled to the brim with water and set in a basin. 1 lb. of gravel is then dropped gently into the jar, and it is found to have displaced 9·4 oz. of water. Find the density of the gravel in lb. per cubic ft and in kilograms per cubic metre.

5. A piece of brass piping has an internal diameter of 1″ and an external diameter of $1\frac{1}{2}$″. Find the weight of 1 ft. of it.

6. A piece of wrought iron plate has a uniform thickness of 2″ and weighs 35 lb. Find its area.

Patterns of Castings.—Many machine parts are made of castings, to obtain which a "pattern" is first made of wood, often yellow pine. This wood weighs about 31 lb. per cubic ft., and since cast iron weighs 450 lb. per cubic ft., it follows that the casting will have a weight $\frac{450}{31}$ ($=14 \cdot 5$) times that of the pattern. In the foundry if the pattern is weighed and then multiplied by the proper factor, we obtain the weight of metal required for the casting. (N.B.—This method is only applicable when the castings are such as do not require a "core." No allowance is made for the shrinkage of the metal, but the method is quite accurate enough for all practical purposes.)

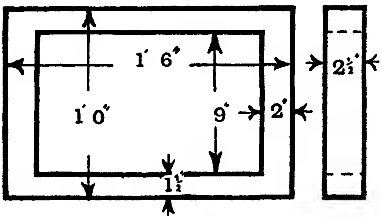

Fig. 67.

Ex. 57. *Fig.* 67 *gives particulars of a pattern. Find the weight of the pattern and of the casting in iron.*

Area enclosed by outside boundary = $(1 \cdot 5' \times 1')$ sq. ft. $= 1 \cdot 5$ sq. ft.

Area of the space in the middle

$$\begin{aligned} &(1'\ 6'' - 4'') \times (1'\ 0'' - 3'') \\ &= (1'\ 2'') \times (9'') \\ &= 1\tfrac{1}{6}' \times \tfrac{3}{4}' \\ &= \tfrac{7}{8} \text{ sq. ft.} = 0 \cdot 875 \text{ sq. ft.} \end{aligned}$$

The area of the upper surface of the pattern is therefore $(1 \cdot 5 - 0 \cdot 875)$ sq. ft. $= 0 \cdot 625$ sq. ft.

Multiplying by the thickness $2\frac{1}{2}'' = \frac{2\frac{1}{2}'}{12} = 0 \cdot 208'$.

Volume $= 0 \cdot 13$ cubic ft.

Yellow Pine has a density of 31 lb. per cubic ft. The pattern therefore weighs $0 \cdot 13 \times 31$ lb. $= 4 \cdot 03$ lb., and the casting in iron weighs

$$4 \cdot 03 \times 14 \cdot 5 = 58 \cdot 5 \text{ lb.}$$

Ex. 58. *Fig.* 68 *shows the diagram of a cutting for a new road. Determine the weight of material removed in cutting a length of* 50 *yards.* (*Take the density of the earth removed as* 125 *lb. per cubic foot.*)

A sectional area of the cutting may be regarded as made up of two right-angled triangles and a rectangle, as shown in Fig. 68.

Area of the rectangle = $15 \times 10 = 150$ sq. ft.
Area of each triangle = $\frac{1}{2}(10 \times 10) = 50$ sq. ft.
,, ,, ,, ,, = ,, 50 ,,
Total sectional area ... 250 ,,

Now the length of the cutting 50 yds. = 150 ft.

Therefore the volume of the material removed = 250×150 cubic ft. and the weight of the material removed = $250 \times 150 \times 125$ lb.

$$= \frac{250 \times 150 \times 125 \text{ tons}}{2240}$$

2100 tons (nearly).

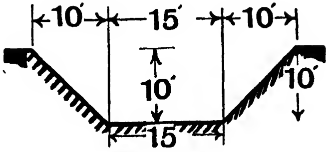

Fig. 68.

Exercises 12b.

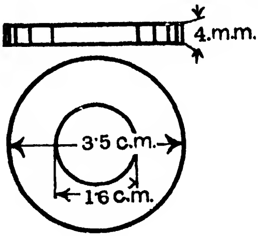

Fig. 69.

1. Determine the factors by which the weight of a yellow pine pattern must be multiplied in order to give the weight of the casting in (1) brass, (2) copper, (3) aluminium.

2. Determine the weight of a length of 50 yards of copper tape connecting a lightning conductor to the earth plate. The copper tape is $1\frac{1}{8}''$ by $\frac{1}{8}''$ in cross section.

3. Calculate the weight of 100 wrought iron washers of the dimensions shown in Fig. 69.

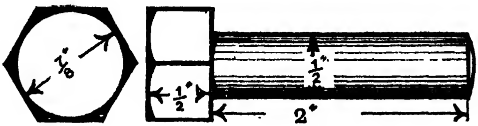

Fig. 70.

4. How much will 50 wrought iron bolts weigh before being screwed if they are made to the sizes shown in Fig. 70?

5. A trench of the cross section shown in Fig. 71 has to be dug for a length of 15′ 0″. If a depth of 2′ 0″ is dug from

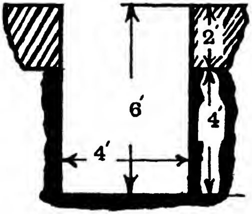

Fig. 71.

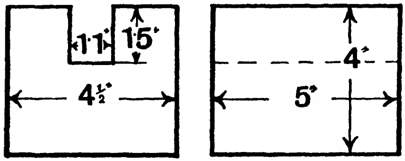

Fig. 72.

common loamy earth weighing 95 lb. per cubic ft. and the remaining 4′ 0″ of depth from clay whose weight is 120 lb. per cubic ft., find how many tons of material are removed.

6. Part of the weight on the weighing lever of a testing machine is shown in Fig. 72. Determine its weight (cast iron). The other portion of the weight, which is not shown

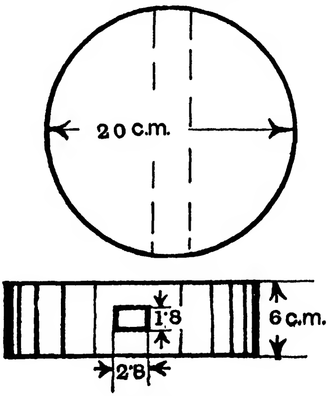

Fig. 73.

on the sketch, weighs 4·5 lb.; what is the ratio of the total weight (calculated + 4·5) to the capacity of the machine (100,000 lb.)?

7. A casting (cast iron) is required of the dimensions shown in Fig. 73. What will it cost if the metal costs 1d. per lb.?

8. The weight sheet of a bridge truss contains the following:—

Length of member in feet.	Dimension of member in cross section.	Weight in lb. per foot of member.	Total weight in cwt. of members.
10·77	4″ by 4″ by $\frac{1}{2}$″ T section	12·8	—
10·77	$3\frac{1}{2}$″ × $3\frac{1}{2}$″ × $\frac{7}{16}$″ T section	9·78	—
12·81	$2\frac{1}{2}$″ × $\frac{3}{8}$″ flat	3·19	—
15·62	3″ × $\frac{7}{16}$″ flat	4·46	—
18·87	$2\frac{1}{2}$″ × $\frac{7}{16}$″ flat	3·72	—

Determine the total weight in cwt. of the sections enumerated in the table.

9. Fig. 74 shows the size of the gudgeon pin for a petrol engine. Determine its weight if the material is mild steel.

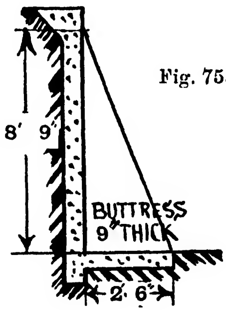

Fig. 75.

10. The buttress of a retaining wall was built in concrete to the dimensions shown in Fig. 75. If it is 9″ thick and

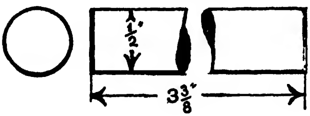

Fig. 74.

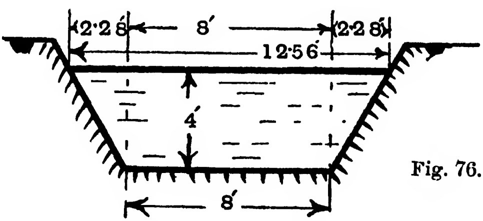

Fig. 76.

is made of concrete weighing 124 lb. per cubic ft., determine the weight in lbs.

11. Fig. 76 shows the section of a cutting for carrying water. If it is 30 ft. long and it is filled with water to the height shown in the figure, determine how many gallons of water it will hold.

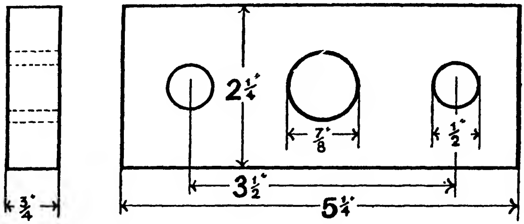

Fig. 77.

12. A small steel casting is required (Fig. 77). Calculate the weight of the pattern (yellow pine), and from this find the weight of 20 of the castings.

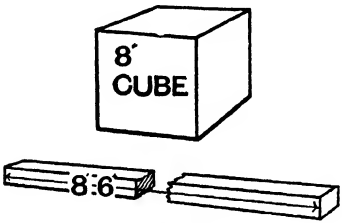

Fig. 78.

13. Determine the weight of a railway sleeper, 10″ by 5″ by 9′ 0″ long. (Take the weight of the material as 31·2 lb. per cubic ft.)

14. Calculate the weight of 100 ft. of bar aluminium of 0·25 in. diameter.

15. An 8 in. cube of wrought iron is heated to a bright red and then flattened out to a square bar (Fig. 78). If the length is 8 ft. 6 in., determine the cross sectional area and the weight of the bar.

CHAPTER XIII.

GRAPHS OF SIMPLE FUNCTIONS.

We have seen that the circumference (C) of a circle of radius r is given by

$$C = 2\pi r$$

Substituting an approximate value for π (3·14) we obtain

$$C = 6{\cdot}28r.$$

Here we have an equation containing two "unknowns," viz. C and r, and we might give any value whatever to r and find what value C would then have. Both these unknowns, then, may be considered as "variables," because their values may be anything. But if we give some definite value to one of them, say r, then the value of C is fixed. We may therefore call r the **independent variable**, in which case C would be referred to as the **dependent variable.** Another way of expressing this same idea is to call C **a function** of r.

Now any equation of this type expresses a relation between the independent and the dependent variables, and, as we have seen in Chapter IV., this relation may be also expressed by means of a graph. In this chapter we shall try to obtain the graph from the equation.

Ex. 59. *Plot the graph of* $C = 6{\cdot}28r$ *for the values of* r *between* 0 *and* 10.

We must now substitute a few values of r into this equation and calculate the corresponding values for C. The values of r may be chosen at random. Thus:—$C = 6{\cdot}28r$.

Therefore when $r = 0$ $C = 6{\cdot}28 \times 0 = 0$,

,, $r = 5$ $C = 6{\cdot}23 \times 5 = 31{\cdot}4$,

,, $r = 10$ $C = 6{\cdot}28 \times 10 = 62{\cdot}8$.

Plotting these three points we find that they lie in a straight line as shown in Fig. 79.

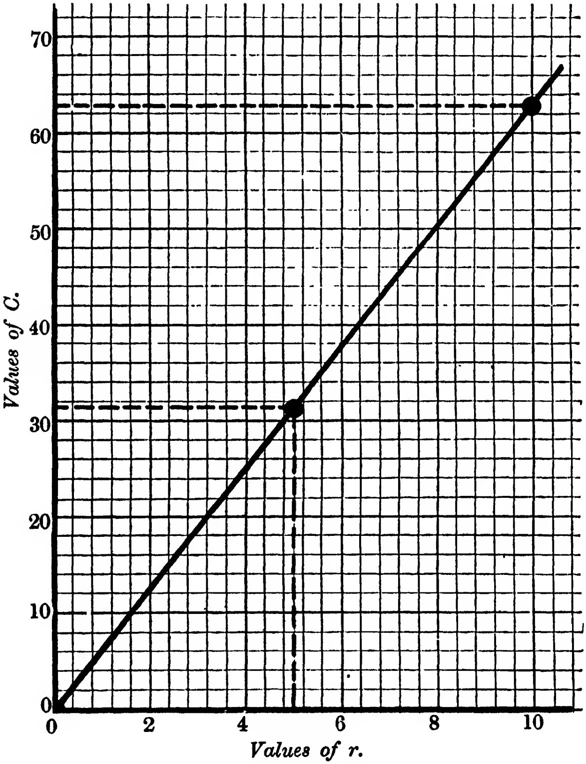

Fig. 79.

It is very convenient when studying the graphs of functions to consider the graphs in families called "types." The simplest

type of graph is a straight line, and this always has an equation of the same type or form. This type is:—

$$y = ax + b,$$

in which y is a function of x, *i.e.* x is the independent and y the dependent variable. Now a and b are constants, *i.e.* they have some definitive numerical values which do not vary. The value of a may be called the "coefficient of x" and that of b the "constant term."

Ex. 60. *Plot the graph of* $y = 2x - 6$.

(In this case the "constant a" equals 2 and the "constant b" equals -6.)

Selecting any two values of x (say 0 and 5) we get $y = 2x - 6$.

When $x = 0$, then $y = 0 - 6 = -6$,

and when $x = 5$, then $y = 10 - 6 = 4$.

We must now go back to our idea of negative direction developed in Chapter IX. Drawing the axes in the shape of a cross, as shown in Fig. 80, the horizontal axis gives the values of x, positive when measured from the origin towards the right and negative when measured towards the left. Similarly the vertical axis gives us positive values of y above the origin and negative values below. The two points taken on the graph are shown in Fig. 80.

Ex. 61. *Plot the graph* $3{\cdot}4x - 2{\cdot}3y = 0{\cdot}704$.

This equation must first be brought into the form of $y = ax + b$.

Thus $3{\cdot}4x - 2{\cdot}3y = 0{\cdot}704$.

Adding $-3{\cdot}4x$ to both sides $-2{\cdot}3y = -3{\cdot}4x + 0{\cdot}704$

Dividing through by $-2{\cdot}3$ $y = \dfrac{3{\cdot}4x - 0{\cdot}704}{2{\cdot}3}$.

Here the constant $a = \dfrac{3{\cdot}4}{2{\cdot}3}$ and the constant $b = -\dfrac{0{\cdot}704}{2{\cdot}3}$.

To obtain two points on the graph take any values of x (say 0 and 2) and substitute them for x thus $y = \dfrac{3{\cdot}4x - 0{\cdot}704}{2{\cdot}3}$.

When $x = 0$ $y = \dfrac{(3{\cdot}4 \times 0) - 0{\cdot}704}{2{\cdot}3}$

$$= \frac{-0{\cdot}704}{2{\cdot}3} = -0{\cdot}306,$$

and when $x = 2 \quad y = \dfrac{(3\cdot4 \times 2) - 0\cdot704}{2\cdot3}$

$$\frac{6\cdot8 - 0\cdot704}{2\cdot3}$$

$$\frac{6\cdot096}{2\cdot3} = 2\cdot64.$$

The student should now plot this graph for himself. A suitable scale would be to let 1 in. represent unity on both axes.

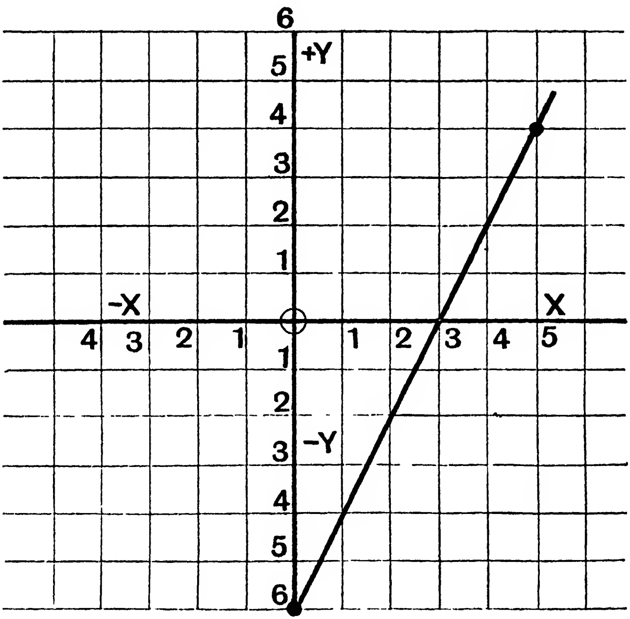

Fig. 80.

In Exercises 10b, No. 19, an equation :—

$$\text{Friction} = 0\cdot2 + 0\cdot0112 \text{ load}$$

was used. This should be recognised as of the type $y = ax + b$; in other words it represents a linear law. Here the independent variable (or x) is "load," and its coefficient (or a) is 0·0112.

The dependent variable (or y) is "friction," and the constant term (or b) is 0·2 ; further, the equation is really written in the order $y = b + ax$, but this of course makes no difference whatever.

It should be noted that in all cases the value of b gives the reading on the "y scale" where the graph cuts that axis. This is obviously so if we regard it as the value of y when $x = 0$.

Exercises 13a.

Plot the graphs of the following equations. Find from the graph the value of y when $x = 0$ and the value of x when $y = 0$.

1. $y = 3x - 3$.

2. $y = 2x + 4$.

3. $y = 3{\cdot}4x - 5{\cdot}2$. Plot each of these exercises from $x = -2$ to $x = 6$.

4. $2x + 3y = 8$.

5. $3x - 2y = 5$.

6. $3{\cdot}2y - 4{\cdot}6x = 1{\cdot}72$. In the last three exercises plot from $x = -5$ to $x = 10$.

7. Plot the graphs of the equations given in Exercises 10b, Nos. 19 and 20, for values of the load between 0 and 50 lb. in both cases. Find by interpolation the loads corresponding to the values of friction there given and compare the results with the calculated values.

8. If H is the number of units of heat required to convert 1 lb. of cold water into steam at $t°$ C., then $H = 606{\cdot}5 + 0{\cdot}305\,t$. Plot a graph giving the value of H between $t = 70°$ C. and $t = 120°$ C. What is the value of the constant H when $t = 100°$ C.

9. When a certain dynamo is running at n revolutions per minute its E.M.F. is v volts. Plot a graph showing the relation between v and n if $v = 0{\cdot}844\,n$, for speeds ranging from 200 to 800 revolutions per minute. What is the voltage when $n = 320$?

10. A steam engine has an indicated horse power of I when it is using W lb. of steam per hour. If $W = 60 + 23{\cdot}75\,I$, plot a graph showing the values of W between $I = 9$ and $I = 30$. How much steam is used per hour when the I.H.P. is $11\frac{1}{2}$?

CHAPTER XIV.

SIMULTANEOUS EQUATIONS.

Methods of Solution.—In Chapter XIII. we considered an equation of the form $7x - 3y = 3$, and we saw that x might have any value whatever, and that y would then have a corresponding value. It follows therefore that a single equation containing two unknowns cannot be solved as were the equations which were considered in Chapter X.

If, however, we were given another equation connecting x and y, such as $5x + 2y = 12{\cdot}5$, which was true at the same time as the first equation, they would be called **simultaneous equations,** and we might find some value for x and y which would satisfy *both* equations.

Example 62. *Solve the following equations, analytically and graphically :—*

$$\text{(i) } 7x - 3y = 3$$
$$\text{(ii) } 5x + 2y = 12{\cdot}5.$$

Analytical Solution.—We must first aim at getting the coefficients of x (*or* y) the same in both equations :—

Equation (i) multiplied throughout by 2............$14x - 6y = 6$
Equation (ii) ,, ,, ,, 3............$15x + 6y = 37{\cdot}5$

Adding..................$29x = 43{\cdot}5$

(N.B.—If the two terms having the same coefficient had both been of the same sign it would have been necessary to *subtract* instead of adding.)

This process is called eliminating one of the unknowns, as we now have an equation with only one unknown (viz. x).

Dividing both terms by 29, we get, $x = \dfrac{43{\cdot}5}{29} = 1\tfrac{1}{2}$.

We can now find y by substituting the value for x into one of the equations, thus:—

(i) $7x - 3y = 3.$

Therefore $(7 \times 1\frac{1}{2}) - 3y = 3,$

i.e. $10\frac{1}{2} - 3y = 3.$

Therefore $10\frac{1}{2} - 3 = 3y.$

$3y = 7\frac{1}{2},$

and $y = 2\frac{1}{2}.$ Result $x = 1\frac{1}{2},\ y = 2\frac{1}{2}.$

The student should test this result by substituting these values into both of the original equations.

Graphical Solution.—To solve these equations graphically we must plot the graphs of both equations.

(i) $7x - 3y = 3.$

Adding $-7x$ to both sides. $-3y = -7x + 3.$

Dividing throughout by -3. $y = \frac{7x - 3}{3}.$

Therefore when $x = 0$. $y = \frac{(7 \times 0) - 3}{3}$

$= -\frac{3}{3} = -1,$

and when $x = 2$. $y = \frac{(7 \times 2) - 3}{3}$

$= \frac{14 - 3}{3}.$

$= \frac{11}{3} = 3\frac{2}{3}.$

(ii) $5x + 2y = 12{\cdot}5.$

Adding $-5x$ to both sides. $2y = -5x + 12{\cdot}5.$

Dividing by 2. $y = \frac{-5x + 12{\cdot}5}{2}.$

When $x = 1$. $y = \frac{(-5 \times 1) + 12{\cdot}5}{2}$

$= \frac{-5 + 12{\cdot}5}{2} = \frac{7{\cdot}5}{2} = 3\frac{3}{4}.$

$x = 3.$ $y = \frac{(-5 \times 3) + 12{\cdot}5}{2}$

$= \frac{-15 + 12{\cdot}5}{2} = -\frac{2{\cdot}5}{2} = -1\frac{1}{4}.$

The solution to the equations is given by the co-ordinates of the point where the graphs intersect. Since this point is the only one which lies on

both graphs, it is obvious that its co-ordinates are the only values of x and y which will satisfy both equations. In the case under consideration the result is $x = 1\frac{1}{2}$, $y = 2\frac{1}{2}$. (See Fig. 81.)

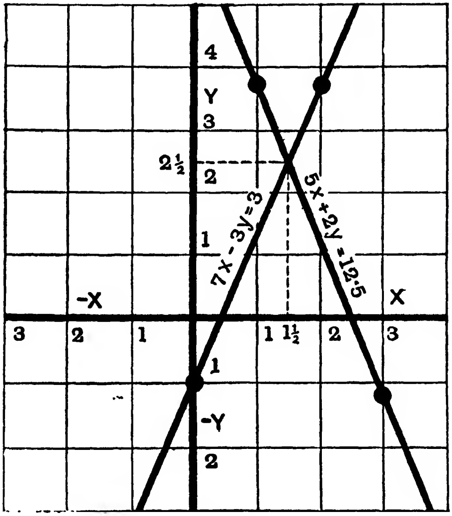

Fig. 81.

Exercises 14a.

Solve the following equations analytically and graphically :—

1. $\left.\begin{aligned} 5x - 2y &= 1 \\ 2x + 3y &= 8 \end{aligned}\right\}$

2. $\left\{\begin{aligned} 3x - y &= 6 \\ 5x - 2y &= 7 \end{aligned}\right.$

3. $\left.\begin{aligned} 4x - 2y &= 11 \\ 7x + 3y &= 29 \end{aligned}\right\}$

4. $\left\{\begin{aligned} 5x - 6y &= 2 \\ 10x - 9y &= 28 \end{aligned}\right.$

5. $\left.\begin{aligned} 8x - 4y &= 9 \\ 4x + 3y &= 20\tfrac{3}{4} \end{aligned}\right\}$

6. $\left\{\begin{aligned} 5x - y &= 0{\cdot}4 \\ 4x + 3y &= 5{\cdot}64 \end{aligned}\right.$

Linear Laws.—We have already met with the results of experiments which when plotted give a straight line graph. We speak of these graphs following a linear law, and it is frequently necessary to express this algebraically.

Example 63. *The following table gives the results of a test on a laboratory crane. L is the load lifted when the handle effort was E. Find the law connecting L and E.*

L (in lb.)	...	0	50	100	150	200
E (in lb.)	...	7·3	8·4	9·4	10·4	11·5

First plot the graph showing the relation between L and E. This is shown in Fig. 82. It will be noticed that these points do not lie *exactly* along a straight line. This does not mean that the linear law is not

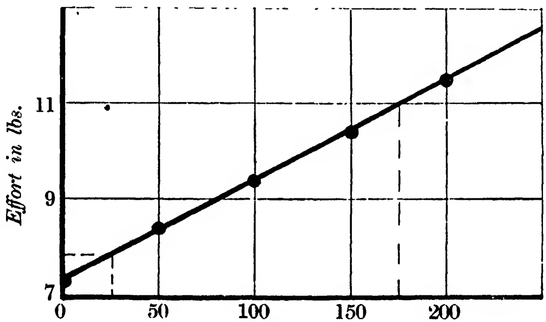

Fig. 82.

followed, but experimental results are always liable to slight error, and, in the case of results obeying linear laws, a straight line should be drawn so as to include as many points as possible and to pass among the remainder.

Now since the graph is a straight line we know that its equation will be of the type $y = ax + b$, *i.e.* using L and E for the two variables we may write $E = aL + b$, and the problem resolves itself into finding values for a and b.

Select two points on the graph, not too close together, and find the values of L and E at each end of them. Thus

(i) When $L = 25, \quad E = 7{\cdot}8.$

(ii) When $L = 175, \quad E = 11.$

Now substitute these values of L and E into the above equation :—

$$E = aL + b.$$

Therefore $\quad 7{\cdot}8 = a \times 25 + b,$

and $\quad 11 = a \times 175 + b.$

Here we have two simultaneous equations with a and b as the unknowns

(i) $\quad 25a + b = 7{\cdot}8$

(ii) $175a + b = 11$

Subtracting $150a \quad = 3{\cdot}2$

Dividing by 150, $a = \dfrac{3{\cdot}2}{150} = 0{\cdot}021.$

(i) $25a + b = 7{\cdot}8.$

Substituting $a = 0{\cdot}021$. $(25 \times 0{\cdot}021) + b = 7{\cdot}8,$

i.e. $0{\cdot}52 + b = 7{\cdot}8.$

Therefore $b = 7{\cdot}3$ (nearly).

Result $a = 0{\cdot}021$ and $b = 7{\cdot}3$, and the required equation is

$$E = 0{\cdot}021\ L + 7{\cdot}3.$$

Exercises 14b.

1. Examine the graph plotted from the data given in Exercise 4*a*, No. 2. Denoting the load by L and the compression of the spring by d, find a law of the type $d = aL + b$.

2. Find the equation of the graph plotted in Exercise 4*a*, No. 3. Calling the Volume of the gas V and its temperature t, then the type will be $V = at + b$.

3. From the graph plotted in Exercise 4*a*, No. 5, find the law connecting the force (F) and the distance (d) which the nail penetrates the wood. The type is $F = ad + b$.

4. The distance across the flats (the two sides upon which the spanner fits), and the diameter of a number of Whitworth nuts and bolts is given.

Diameter of the bolt in inches ...	$\frac{1}{2}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$
Distance across the flats	$\frac{7}{8}$	$1\frac{5}{8}$	2	$2\frac{3}{8}$

Plot the graph and find the equation. It is of the type:—
Distance across the flats $(y) = b + a \times$ diameter of bolt (x), and this equation is very useful for drawing Whitworth nuts and bolt heads.

5. The price of a gross of wrought iron pins, $\frac{3}{4}''$ diameter is given below for different lengths.

Length in inches (l) ...	$7\frac{1}{2}$	6	$4\frac{1}{2}$	3
Price (P) per gross ...	55s. 6d.	45s. 6d.	36s.	27s.

Find the law of the type $P = al + b$.

6. The following figures were obtained from lifts of the same size :—

Total annual maintenance cost including interest and depreciation	£40	£60	£80	£100
Total journeys per year ...	20,000	40,000	60,000	80,000

Plot the graph and find a law of the type,

$$\text{Cost} = a \times \text{journeys per year} + b.$$

7. The following table shows how the width of a sunk key (used for fastening a wheel to the shaft) varies with the diameter of the shaft :—

Diameter of the shaft in inches ...	1	2	3	4
Width of key in in.	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$

Prove that the law is :—

$$\text{Width of the key}'' = \frac{\text{Diameter of shaft}''}{4} + \tfrac{1}{8}''.$$

ANSWERS.

Exercises 1a.

2. 4·51″. **3.** 6, 4·4, 3·4, 1·9 cm. **4.** 5·01″, 0·99″.

5. $\frac{1}{16}''$, $\frac{3}{16}''$, $\frac{5}{16}''$, $\frac{1}{2}''$, $\frac{5}{8}''$, $\frac{3}{4}''$, $\frac{7}{8}''$. **6.** ·83″, 2·01″, 1″, 4″, 4·5″.

Exercises 1b.

1. 4·4″, 112 mm. **2.** 3·3″, $3\frac{1}{4}''$. **3.** 143·4 mm., 5·64″.

4. 51·976, 52, 24·8 mm. **5.** 4·305 ft., 4·3 ft. **6.** 1209·53 sq. ft.

Exercises 1c.

1. 5·93. **2.** 1100. **3.** 10·8, 0·2, 11·6, 8·8, 971·7, 0·9.

4. 1·16″, 2·95 cm. **5.** 139 mm., 5·48″. **6.** 5359.

Exercises 1d.

1. 2411, 6·35, 9·87, 5·97, 16·93. **2.** 445·4, 13·9, 87·5, 1·0, 11·9, 381·9.

3. ·169, ·0824, ·0353. **4.** 9·99″, 1″, 78·59″, ·38″.

5. 497·9″, 109·8″, 263·8″, 37·4″. **6.** 41·7, 312·0, 1·7, 135·2.

Exercises 1e.

1. ·5, ·25, ·125, ·0625, ·01. **2.** ·03125, ·04, ·025, ·001, ·015625.

3. 4·7, 33·4, 86·1, ·8. **4.** 250, 128, 456·7, ·004, ·0078, ·00219.

5. 65·1, ·478, ·01. **6.** 3·143, ·786, 24·4, 53·9, ·00837.

Exercises 1f.

1. 2·875″. **2.** 16·35 cm., 6·44″. **3.** 4·24″, 10·78 cm.

4. 47·9, 17·1, 342, 1·206. **5.** 90·85, 17·85. **6.** 103·3.

Exercises 2a.

1. $\frac{13}{16}$. **2.** $\frac{29}{32}$. **3.** $\frac{23}{24}$. **4.** $2\frac{9}{16}$. **5.** $2\frac{27}{32}$. **6.** $3\frac{33}{64}$.

Exercises 2b.

1. $\frac{23}{32}$. **2.** $2\frac{1}{4}$. **3.** $1\frac{19}{32}$. **4.** 14. **5.** $2\frac{3}{16}$. **6.** $6\frac{47}{68}$.

Exercises 2c.

1. 1·75. **2.** 2·625. **3.** 2·417 (nearly). **4.** 1·4375.
5. 3·40625. **6.** 2·391 (nearly).

Exercises 2d.

1. $\frac{1}{4}$. **2.** $1\frac{1}{8}$. **3.** $2\frac{2}{3}$. **4.** $\frac{1}{16}$. **5.** $1\frac{7}{16}$. **6.** $\frac{9}{32}$.

Exercises 2e.

1. 1·75. **2.** $3{\cdot}\dot{3}$. **3.** $6\frac{1}{3}$. **4.** $\frac{3}{4}$. **5.** $\frac{4}{7}$. **6.** 5″.

Exercises 3a.

1. 6·45 sq. cm. **2.** 1550 sq. in. **3.** 13 sq. ft.
4. 16·4 c.c. **5.** 35·3 cub. ft. **6.** 3818 c.c.

Exercises 3b.

1. 1·414, 1·732, 2·236, 2·646. **2.** 5·656, 8·660, 5·292. **3.** 253.
4. 22·96. **5.** 70. **6.** 22·9 cm.

Exercises 4a.

1. 121·5 c.c., 6·1 cub. in. **2.** ·175″, ·44″, ·81″, 7·1 lb.
3. 13·7 c.c., 69° C. **4.** 26 lb., 39 lb.
5. 214 lb., 277 lb. **6.** 77° F., 82° C.
7. $5\frac{1}{4}$d., $8\frac{1}{4}$d.. $11\frac{1}{4}$d. **8.** 29s. 6d., 41s. 6d., 56s. 6d., 68s. 6d.
9. 35·9 lb., 61·4 lb. **10.** 1·42 sec., 24·8 cm.

Exercises 5a.

1. 2·54, ·394, ·16, 16·4, ·093. **2.** ·275.
3. ·0556, ·1, ·111, ·125, ·167, ·2, ·222. **4.** 3·14.
5. $\frac{3}{19}$, $\frac{1}{5}$. **6.** 1·414.
7. ·0485″ per inch, ·582″ per foot.
8. ·162″ per foot, 4·12 mm. per 30·48 cm. **9.** 29·7 sq. ft.
10. $\frac{1}{2}$. **11.** 1·13 per cent., 18·5 per cent.
12. £1 13s. 4d., 12s. **13.** $51\frac{1}{4}$ per cent.
14. 9s., £315. **15.** 618 lb. per sq. in., 257 lb. per sq. in.
16. 2·6″. **17.** ·83″. **18.** ·287″.

Exercises 6a.

1. $A = 35°$, $B = 90°$, $C = 55°$, 46·1 cm.
2. $A = 63°$, $B = 39°$, $C = 78°$, 9·69″.
3. c = 15·35 cm., $A = 41°$, $B = 84°$, 46·29 cm.
4. b = 6·38′, $A = 36\frac{1}{2}°$, $C = 88\frac{1}{2}°$, 18·84″.
5. a = 5·8″, $A = 50°$, $C = 72°$, 19·6″.
6. b = 18·7 cm., $A = 42°$, $B = 90°$, 45 cm.

Exercises 6b.

1. 21·2 sq. cm. **2.** 2·12 sq. in. **3.** 30·2 sq. cm.
4. 5·5 sq. in. **5.** 76·3. **6.** 75·8.

Exercises 6c.

1. 74 ft. 3 rolls. **2.** 352 bricks. **3.** 18·96 sq. ft. **4.** 702 sq. in.
5. 24 sq. in. Each space should be 6″ high and 4″ wide.
6. 1·08 acres, 4320 sq. metres.

Exercises 7a.

1.

Areas.	Circumferences.
sq. cm.	cm.
·126	·126
1·54	4·4
5·31	8·17
18·2	15·1
41·9	22·93

2.

Circumferences.	Areas.
in.	sq. in.
·189	·00282
2·54	·515
7·73	4·75
15·02	17·9
16·56	21·8

3.

Diameters.	Areas.
in.	sq. in.
1	·785
5·49	23·7
7·40	43·0
24·1	456
36·8	1062

Exercises 7b.

1. ·00246 sq. in., 1·586 sq. mm., ·0000283 sq. in., ·0183 sq. mm.
2. 132·8 sq. cm. **3.** 17·3 ft., 21·2″. Between 305 and 306.
4. Distance from A to B = 3·33 ft., 20 strips., 22 ft. approximately.
5. ·00306 sq. ft., 2·84 sq. cm. **6.** At A ·993 sq. in., at B 2·4 sq. in.
7. AB = 53·8 ft., 11740 lb. **8.** A = 28·3 ft., B = 32 ft.
9. 3·93 measures round the circle and 3·75 measured in a straight line from hole to hole.

Exercises 8a.

1. ·267 sq. in. **2.** 4·87″, ·72″, 3·88″ sq. in.* **3.** ·441 sq. in., ·0275 cub. in.
4. ·281. **5.** $\frac{7}{8}$, $\frac{15}{16}$, 1, $1\frac{1}{8}$.* **6.** 13·9.
7. 55·9 ft. **8.** 5·15. **9.** 14·7 sq. cm.
10. 6·9 ohms. **11.** 4·3 sq. cm. **12.** ·54″, 1·22″, 2·16″.

Exercises 9a.

1. $19x + 21y + 11z$, $19a + 13b + 14$.
3. $12x + 15y + 11$, $10S + 13W + 12$.
5. $\frac{13}{16}a + 7b + 6c$, 9·55. **6.** $13\frac{1}{3}$ shillings, $25\frac{1}{8}$ shillings.

Exercises 9b.

2. $3a + 9b - 2c$, $a - 6b + 7c$, $2b$. **3.** $5y$, $-2W - 10X + 2Z$.
5. $-0{\cdot}5x^2 - 4xy - 3y^2$, $-7x^3 - 3Q^2 - 8p$.
6. $-a - 7b + 6c$, $3a - 6x + y$.

Exercises 9c.

1. $12bx$. **2.** $15x^2y$. **3.** $12p^2q$. **4.** $-30ab^2$.
5. $-4x^2y^2$. **6.** $2a^2bc^2$. **7.** $2a^2 + 5a - 12$. **8.** $x^2 - x - 56$.
9. $4x^2 - 16$. **10.** $3x^3 - 12x^2 + 6x$.
11. $2a^3 - 6ab^2 + 4b^3$. **12.** $10p^3 - 9p^2q + 8pq^2 - 3q^3$.

Exercises 9d.

1. $5xy$. **2.** $4x^2$. **3.** $3qr$. **4.** $x - 2$.
5. $x + 3$. **6.** $3x - 2$. **7.** $p^2 + 4pq + 4q^2$. **8.** $a^2 - 6ab + 9b^2$.
9. $4x^2 - 4xy + y^2$. **10.** $a^3 + 3a^2y + 3ay^2 + y^3$.
11. $8p^3 - 12p^2q + 6pq^2 - q^3$. **12.** $a^3 - 6a^2b + 12ab^2 - 8b^3$.
13. $(x + 8)(x - 8)$. **14.** $(a - 3)^2$. **15.** $(x + 4)^2$.
16. $(x - 2)(x^2 + 2x + 4)$. **17.** $(a + 3)(a^2 - 3a + 9)$.
18. $(4 - p)(16 + 4p + p^2)$.

Exercises 9e.

2. $4a - 5b + 5c$. **3.** $6a - 1{\cdot}5b - 6x$.
4. 18 files, 80 screwdrivers, 32 cold chisels.

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* In practice stock sizes must be considered. Thus in No. 2, 4·87 would become $4\frac{7}{8}$ and 0·72 would be considered as $\frac{3}{4}$. Similarly in No. 5: rivets of $\frac{15}{16}$″ diameter are not made, and 1″ rivets would have to be used.

5. $\pi(w^2 + l^2 + y^2)$, $\frac{21\pi}{16}$. **6.** $\frac{ln}{33000}(P + q)$, $1{\cdot}105A\sqrt{2gh}$.

7. $2\pi b^2 - 8\pi c^2$. **9.** $-a - 7b + 6c$, $3a - 6x + y$.

10. $2ax - 10bx - ab$. **11.** $-\frac{5\pi D^2}{4} + \frac{11\pi d^2}{4}$, $dQa - 2Qb + Pa$.

12. $-0{\cdot}5x^2 - 4xy - 3y^2$, $-7x^3 - 3Q^2 - 8p$. **13.** $13{\cdot}25$.

14. $6ax + 18ay + 6az$, $3x^3 + 9x^2y + 3x^2z$. **15.** $25x^2$, $6\frac{1}{4}$.

16. $x^2 + 4px + 4p^2$, $2x^2 + 9px + 9p^2$, $x^3 - 3x^2p + 3xp^2 - 3p^3$.

17. $3{\cdot}26D^2n^2 - 3{\cdot}46Dn + 0{\cdot}2$. **18.** $-x^3 - 2x^2 + 41x - 30$.

19. 2, $2a + b$, $x + y$. **20.** $\frac{rp}{2t}$, $\frac{1}{2}(a + b)$, $\frac{3(a^2 + ab + b^2)}{(a + b)}$.

21. $a + 2b$. **22.** $\frac{\pi(D^2 + Dd + d^2)}{32}$, $\frac{11l}{8}$.

23. $\frac{H^3 + 4}{30H}$. **24.** $x - b$, $x - \frac{1}{3}$, $a - b - c$.

Exercises 10a.

1. $4\frac{1}{8}$. **2.** 15. **3.** 1. **4.** 1·8. **5.** $3\frac{1}{2}$.
6. 2. **7.** 2. **8.** $6\frac{3}{4}$. **9.** $7\frac{1}{2}$.

Exercises 10b.

1. 21 volts. **2.** 5·77 ohms. **3.** 1·69 ampères.
4. 21·8° C. **5.** 25·3 in. **6.** 1·6″, 2·67″, 4·61″.
7. 155·3 ft. **8.** 176° C. **9.** $4\frac{1}{2}$ lb.
10. $15\frac{3}{4}$ ft. **11.** 1·22 ft. **12.** 23 tons.
13. 0·4 sq. in., 7 tons. **14.** 62·4 lb. per sq. ft., 5·2 lb. per sq. ft., 2·31 ft.
15. 30300000, ·00094. **16.** 1·55 ft., 3·5 ft., 5·03 ft., 6·21 ft.
17. 1·43″, 1·6″, 1·75″, 2·47″. **18.** $1\frac{7}{8}$.
19. 16·06 lb., 17·85 lb., 19·64 lb., 22·3 lb.
20. 18·35 lb., 24·86 lb., 45·7 lb. **21.** 203, 172, 140·6.
22. 50. **23.** $2\frac{1}{2}$ ft. **24.** 85, 1321, 781, 687.
25. 23·86.

Exercises 11a.

1. 6, 12, 8, 24 sq. in., 8 cub. in. **2.** 22 sq. in., 6 cub. in.
3. 110·6 sq. cm., 52·2 c.c. **4.** 3·54 sq. in., 14·14 sq. in., 5·3 cub. in.
5. 5·65 cm., 12·6 sq. cm., 37·7 sq. cm., 23·6 c.c.

Exercises 11b.

1. 57 sq. ft., 3 sheets, 36 cub. ft.
2. 1775 watts, 7 lamps nearly sufficient.
3. $5947\frac{1}{2}$ sq. ft., 1190, 14869 cub. ft.

4. $6\frac{1}{3}$ cub. ft., 39·52 gallons, 40 gallons.
5. 1665 cub. ft , 195 sq. ft., 4s.
6. 50) cub. ft., 375 cub. ft.
7. 169·5 c.c., 169·5 sq. cm.
8. 0·442 sq. in.
9. 2 sheets, $\frac{5}{8}$ sq. ft. extra required.
10. 579 c.c.
11. 28·3″, 31·4″, 34·5″, 652 cub. in., 237 cub. in.
12. 179 c.c.

Exercises 12a.

1. 97·5 gallons, 1020 lb.
2. 6933 c.c., 8818 c.c.
3. 42 lb. per cub. ft., 0·67 gm. per c.c.
4. 106·4 lb. per cub. ft., 1699 kilogm. per cub. metre.
5. 3·7 lb.
6. 63 sq. in.

Exercises 12b.

1. 17·5, 17·8, 5·2.
2. 80·4 lb.
3. 2·34 kilogm.
4. 10·2 lb.
5. 18 tons (nearly).
6. 21·3 lb., 0·00026.
7. 2s. $4\frac{1}{2}$d.
8. 3·8 cwt.
9. $3\frac{1}{4}$ oz.
10. 1018 lb.
11. 7700 gallons.
12. 2·3 oz., and 46 lb.
13. 97·5 lb.
14. $5\frac{1}{2}$ lb.
15. 5·02 sq. in., 142 lb.

Exercises 13a.

1. -3, 1.
2. 4, -2.
3. $-5·2$, 1·53.
4. $2\frac{2}{3}$, 4.
5. $-2\frac{1}{2}$, $1\frac{2}{3}$.
6. 0·54, $-0·375$.
7. 16·06, 17·85, 19·64, 22·3.
18·35, 24·86, 45·7.
8. 637.
9. 270 volts.
10. 333.

Exercises 14a.

1. $x = 1$, $y = 2$.
2. $x = 5$, $y = 9$.
3. $x = 3\frac{1}{2}$, $y = 1\frac{1}{2}$.
4. $x = 10$, $y = 8$.
5. $x = 2\frac{3}{4}$, $y = 3\frac{1}{4}$.
6. $x = ·36$, $y = 1·4$.

Exercises 14b.

1. $d = ·07L$.
2. $V = ·045t + 12·4$.
3. $F = 252d - 6$.
4. Distance = 1·5 (diameter) + ·125.
5. $P = 6·3l + 8$.
6. Cost = ·001 (no. of journeys) + 20.